1. Operations with Real Numbers
    1. Use the Language of Algebra
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Use the Language of Algebra

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Simplify expressions using the order of operations
- Evaluate an expression
- Identify and combine like terms
- Translate an English phrase to an algebraic expression

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **The Language of Algebra**.

Use Variables and Algebraic Symbols

Suppose this year Greg is 20 years old and Alex is 23. You know that Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are **variables** and the 3 is a **constant**. The ages change ("vary") but the 3 years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by 3 years, 3 is the *constant*.

In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age g, then we could use $g+3$ to represent Alex's age. See [link].

Greg's age	**Alex's age**
12	15
20	23
35	38
$\boldsymbol{g}$	$\boldsymbol{g}+3$

The letters used to represent these changing ages are called *variables*. The letters most commonly used for variables are *x*, *y*, *a*, *b*, and *c*.

Note:
Variable
A **variable** is a letter that represents a number whose value may change.

Note:
Constant
A **constant** is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic operations: addition, subtraction, multiplication, and division. We’ll list the symbols used to indicate these operations below ([link]). You’ll probably recognize some of them.

Operation	**Notation**	**Say:**	**The result is…**
Addition	$a+b$	*a* plus *b*	the sum of *a* and *b*
Subtraction	$a-b$	*a* minus *b*	the difference of *a* and *b*
Multiplication	$a\cdot b, ab, (a)\,(b), (a)b, a\,(b)$	*a* times *b*	the product of *a* and *b*
Division	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$	*a* divided by *b*	the quotient of *a* and *b*, *a* is called the dividend, and *b* is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words “of” and “and.”

- The *difference of* 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9-2$.
- The *product of* 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as $4\cdot 8$.

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does 3*xy* mean $3\ \times\ y$ (‘three times *y*’) or $3\cdot x\cdot y$ (three times *x* times *y*)? To make it clear, use · or parentheses for multiplication.

When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

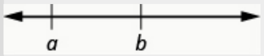
Note:
Equality Symbol
$a=b$ is read “*a* is equal to *b*”
The symbol “$=$” is called the **equal sign**.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols “<” and “>.”

Note:
Inequality

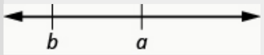
Equation:

$$a<b \text{ is read “}a\text{ is less than }b\text{”}$$
$$a \text{ is to the left of } b \text{ on the number line}$$



Equation:

$$a>b \text{ is read “}a\text{ is greater than }b\text{”}$$
$$a \text{ is to the right of } b \text{ on the number line}$$



The expressions *a* < b or a > *b* can be read from left to right or right to left, though in English we usually read from left to right ([link]). In general, *a* < b is equivalent to b > *a*. For example 7 < 11 is equivalent to 11 > 7. And *a* > *b* is equivalent to *b* < a. For example 17 > 4 is equivalent to 4 < 17.

Inequality Symbols	Words
$a \neq b$	*a* is *not equal to b*
a < b	*a* is *less than b*
$a \leq b$	*a* is *less than or equal to b*
a > *b*	*a* is *greater than b*
$a \geq b$	*a* is *greater than or equal to b*

Example:
Exercise:

Problem: Translate from algebra into English:

ⓐ $17 \leq 26$ ⓑ $8 \neq 17-8$ ⓒ $12>27 \div 3$ ⓓ $y+7<19$

Solution:
Solution

ⓐ $17 \leq 26$
17 is less than or equal to 26

ⓑ $8 \neq 17 - 8$
8 is not equal to 17 minus 8

ⓒ $12 > 27 \div 3$
12 is greater than 27 divided by 3

ⓓ $y + 7 < 19$
y plus 7 is less than 19

Note:
Exercise:

Problem: Translate from algebra into English:

ⓐ $14 \leq 27$ ⓑ $19 - 2 \neq 8$ ⓒ $12 > 4 \div 2$ ⓓ $x - 7 < 1$

Solution:

ⓐ 14 is less than or equal to 27 ⓑ 19 minus 2 is not equal to 8 ⓒ 12 is greater than 4 divided by 2 ⓓ *x* minus 7 is less than 1

Note:
Exercise:

Problem: Translate from algebra into English:

ⓐ $19 \geq 15$ ⓑ $7 = 12 - 5$ ⓒ $15 \div 3 < 8$ ⓓ $y + 3 > 6$

Solution:

ⓐ 19 is greater than or equal to 15 ⓑ 7 is equal to 12 minus 5 ⓒ 15 divided by 3 is less than 8 ⓓ *y* plus 3 is greater than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

Note:
Grouping Symbols
Equation:

$$\begin{array}{ll} \text{Parentheses} & () \\ \text{Brackets} & [] \\ \text{Braces} & \{\} \end{array}$$

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.

Equation:

$$8(14-8) \qquad 21-3[2+4(9-8)] \qquad 24\div\{13-2[1(6-5)+4]\}$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have *expressions* and *equations*.

Note:
Expression
An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Expression	**Words**	**English Phrase**
$3+5$	3 plus 5	the sum of three and five
$n-1$	*n* minus one	the difference of *n* and one
$6\cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	*x* divided by *y*	the quotient of *x* and *y*

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb.

An **equation** is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Note:
Equation
An **equation** is two expressions connected by an equal sign.

Here are some examples of equations.

Equation	**English Sentence**
$3+5=8$	The sum of three and five is equal to eight.
$n-1=14$	*n* minus one equals fourteen.
$6\cdot 7=42$	The product of six and seven is equal to forty-two.
$x=53$	*x* is equal to fifty-three.
$y+9=2y-3$	*y* plus nine is equal to two *y* minus three.

Example:
Exercise:

Problem: Determine if each is an expression or an equation:

ⓐ $2(x+3)=10$ ⓑ $4(y-1)+1$ ⓒ $x\div 25$ ⓓ $y+8=40$

Solution:
Solution

ⓐ $2(x+3)=10$	This is an $equation$—two expressions are connected with an equal sign.
ⓑ $4(y-1)+1$	This is an $expression$—no equal sign.
ⓒ $x\div 25$	This is an $expression$—no equal sign.
ⓓ $y+8=40$	This is an $equation$—two expressions are connected with an equal sign.

Note:
Exercise:

Problem: Determine if each is an expression or an equation: ⓐ $3(x-7)=27$ ⓑ $5(4y-2)-7$.

Solution:

ⓐ equation ⓑ expression

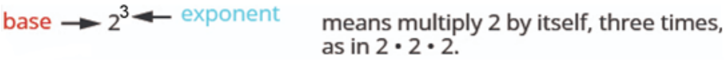
Note:
Exercise:

Problem: Determine if each is an expression or an equation: ⓐ $y^3\div 14$ ⓑ $4x-6=22$.

Solution:

ⓐ expression ⓑ equation

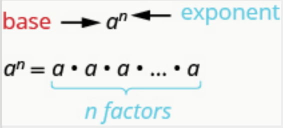
Suppose we need to multiply 2 nine times. We could write this as $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. This is tedious and it can be hard to keep track of all those 2s, so we use exponents. We write $2 \cdot 2 \cdot 2$ as 2^3 and $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^9. In expressions such as 2^3, the 2 is called the *base* and the 3 is called the *exponent*. The exponent tells us how many times we need to multiply the base.



We read 2^3 as “two to the third power” or “two cubed.”

We say 2^3 is in *exponential notation* and $2 \cdot 2 \cdot 2$ is in *expanded notation*.

Note:
Exponential Notation
a^n means multiply *a* by itself, *n* times.


base
a^n
exponent
$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$

The expression a^n is read *a* to the n^{th} power.

While we read a^n as “*a* to the n^{th} power,” we usually read:

- a^2 “*a* squared”
- a^3 “*a* cubed”

We’ll see later why a^2 and a^3 have special names.

[link] shows how we read some expressions with exponents.

Expression	In Words
7^2	7 to the second power or 7 squared
5^3	5 to the third power or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

Example:
Exercise:

Problem: Simplify: 3^4.

Solution:
Solution

	3^4
Expand the expression.	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right.	$9 \cdot 3 \cdot 3$
Multiply.	$27 \cdot 3$
Multiply.	81

Note:
Exercise:

Problem: Simplify: ⓐ 5^3 ⓑ 1^7.

Solution:

ⓐ 125 ⓑ 1

Note:
Exercise:

Problem: Simplify: ⓐ 7^2 ⓑ 0^5.

Solution:

ⓐ 49 ⓑ 0

Simplify Expressions Using the Order of Operations

To **simplify an expression** means to do all the math possible. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:
Equation:

$$\begin{array}{c} 4 \cdot 2 + 1 \\ 8 + 1 \\ 9 \end{array}$$

By not using an equal sign when you simplify an expression, you may avoid confusing expressions with equations.

Note:

Simplify an Expression
To **simplify an expression**, do all operations in the expression.

We’ve introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values. For example, consider the expression:

Equation:

$$4+3\cdot 7$$

If you simplify this expression, what do you get?

Some students say 49,

Equation:

$$\begin{array}{ll} & 4+3\cdot 7 \\ \text{Since } 4+3 \text{ gives 7.} & 7\cdot 7 \\ \text{And } 7\cdot 7 \text{ is 49.} & 49 \end{array}$$

Others say 25,

Equation:

$$\begin{array}{ll} & 4+3\cdot 7 \\ \text{Since } 3\cdot 7 \text{ is 21.} & 4+21 \\ \text{And } 21+4 \text{ makes 25.} & 25 \end{array}$$

Imagine the confusion in our banking system if every problem had several different correct answers!

The same expression should give the same result. So mathematicians early on established some guidelines that are called the Order of Operations.

Note:
Perform the Order of Operations.

Parentheses and Other Grouping Symbols
- Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.

Exponents
- Simplify all expressions with exponents.

Multiplication and Division
- Perform all multiplication and division in order from left to right. These operations have equal priority.

Addition and Subtraction
- Perform all addition and subtraction in order from left to right. These operations have equal priority.

Note: Doing the Manipulative Mathematics activity “Game of 24” give you practice using the order of operations.

Students often ask, “How will I remember the order?” Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase: “Please Excuse My Dear Aunt Sally.”
Equation:

Parentheses	**P**lease
Exponents	**E**xcuse
Multiplication **D**ivision	**M**y **D**ear
Addition **S**ubtraction	**A**unt **S**ally

It’s good that “**M**y **D**ear” goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, “**A**unt **S**ally” goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

Let’s try an example.

Example:
Exercise:

Problem: Simplify: ⓐ $4+3\cdot 7$ ⓑ $(4+3)\cdot 7$.

Solution:
Solution

ⓐ

	$4+3\cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4+3\cdot 7$
Add.	$4+21$

	25

ⓑ

	$(4+3)\cdot 7$
Are there any **p**arentheses? Yes.	$(4+3)\cdot 7$
Simplify inside the parentheses.	$(7)7$
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

Note:
Exercise:

Problem: Simplify: ⓐ $12-5\cdot 2$ ⓑ $(12-5)\cdot 2$.

Solution:

ⓐ 2 ⓑ 14

Note:
Exercise:

Problem: Simplify: ⓐ $8+3\cdot 9$ ⓑ $(8+3)\cdot 9$.

Solution:

ⓐ 35 ⓑ 99

Example:
Exercise:

Problem: Simplify: $18 \div 6 + 4\left(5 - 2\right)$.

Solution:
Solution

Parentheses? Yes, subtract first.	$18 \div 6 + 4\left(5 - 2\right)$ $18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	$18 \div 6 + 4(3)$
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

Note:
Exercise:

Problem: Simplify: $30 \div 5 + 10\left(3 - 2\right)$.

Solution:

16

Note:
Exercise:

Problem: Simplify: $70 \div 10 + 4\left(6 - 2\right)$.

Solution:

23

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

Example:
Exercise:

Problem: Simplify: $5+2^3+3\left[6-3\left(4-2\right)\right]$.

Solution:
Solution

	$5+2^3+3[6-3(4-2)]$
Are there any parentheses (or other grouping symbol)? Yes.	
Focus on the parentheses that are inside the brackets.	$5+2^3+3[6-3(4-2)]$
Subtract.	$5+2^3+3[6-3(2)]$
Continue inside the brackets and multiply.	$5+2^3+3[6-6]$
Continue inside the brackets and subtract.	$5+2^3+3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	$5+2^3+3[0]$
Simplify exponents.	$5+8+3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5+8+0$

Is there any addition or subtraction? Yes.	
Add.	$13 + 0$
Add.	13

Note:
Exercise:

Problem: Simplify: $9 + 5^3 - [4(9 + 3)]$.

Solution:

86

Note:
Exercise:

Problem: Simplify: $7^2 - 2[4(5 + 1)]$.

Solution:

1

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we’ll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

Note:
Evaluate an Expression
To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Example:

Exercise:

Problem: Evaluate $7x - 4$, when ⓐ $x = 5$ and ⓑ $x = 1$.

Solution:
Solution

ⓐ

when $x = 5$	$7x - 4$
	$7(5) - 4$
Multiply.	$35 - 4$
Subtract.	31

ⓑ

when $x = 1$	$7x - 4$
	$7(1) - 4$
Multiply.	$7 - 4$
Subtract.	3

Note:
Exercise:

Problem: Evaluate $8x - 3$, when ⓐ $x = 2$ and ⓑ $x = 1$.

Solution:

ⓐ 13 ⓑ 5

Note:
Exercise:

Problem: Evaluate $4y - 4$, when ⓐ $y = 3$ and ⓑ $y = 5$.

Solution:

ⓐ 8 ⓑ 16

Example:
Exercise:

Problem: Evaluate each expression when $x = 4$: ⓐ x^2 ⓑ 3^x.

Solution:
Solution

ⓐ

	x^2
Replace *x* with 4.	4^2
Use definition of exponent.	$4 \cdot 4$
Simplify.	16

ⓑ

	3^x
Replace *x* with 4.	3^4

Use definition of exponent.	$3 \cdot 3 \cdot 3 \cdot 3$
Simplify.	81

Note:
Exercise:

Problem: Evaluate each expression when $x = 3$: ⓐ x^2 ⓑ 4^x.

Solution:

ⓐ 9 ⓑ 64

Note:
Exercise:

Problem: Evaluate each expression when $x = 6$: ⓐ x^3 ⓑ 2^x.

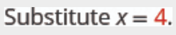
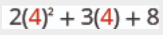
Solution:

ⓐ 216 ⓑ 64

Example:
Exercise:

Problem: Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution:
Solution

	$2x^2 + 3x + 8$
 Substitute $x = 4$.	 $2(4)^2 + 3(4) + 8$
Follow the order of operations.	$2(16) + 3(4) + 8$
	$32 + 12 + 8$
	52

Note:
Exercise:

Problem: Evaluate $3x^2 + 4x + 1$ when $x = 3$.

Solution:

40

Note:
Exercise:

Problem: Evaluate $6x^2 - 4x - 7$ when $x = 2$.

Solution:

9

Identify and Combine Like Terms

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

Note:
Term
A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are $7, y, 5x^2, 9a,$ and b^5.

The constant that multiplies the variable is called the **coefficient**.

Note:
Coefficient
The **coefficient** of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x = 1 \cdot x$.

Example:

Exercise:

Problem: Identify the coefficient of each term: ⓐ $14y$ ⓑ $15x^2$ ⓒ a.

Solution:
Solution

ⓐ The coefficient of $14y$ is 14.

ⓑ The coefficient of $15x^2$ is 15.

ⓒ The coefficient of a is 1 since $a = 1a$.

Note:
Exercise:

Problem: Identify the coefficient of each term: ⓐ $17x$ ⓑ $41b^2$ ⓒ z.

Solution:

ⓐ 17 ⓑ 41 ⓒ 1

Note:
Exercise:

Problem: Identify the coefficient of each term: ⓐ $9p$ ⓑ $13a^3$ ⓒ y^3.

Solution:

ⓐ 9 ⓑ 13 ⓒ 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?
Equation:

$$5x \qquad 7 \qquad n^2 \qquad 4 \qquad 3x \qquad 9n^2$$

The 7 and the 4 are both constant terms.

The *5x* and the 3*x* are both terms with *x*.

The n^2 and the $9n^2$ are both terms with n^2.

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- 5*x* and 3*x* are like terms.
- x^2 and $9x^2$ are like terms.

Note:
Like Terms
Terms that are either constants or have the same variables raised to the same powers are called **like terms**.

Example:
Exercise:

Problem: Identify the like terms: y^3, $7x^2$, 14, 23, $4y^3$, $9x$, $5x^2$.

Solution:
Solution

y^3 and $4y^3$ are like terms because both have y^3; the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2; the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like $9x$.

Note:
Exercise:

Problem: Identify the like terms: 9, $2x^3$, y^2, $8x^3$, 15, $9y$, $11y^2$.

Solution:

9 and 15, y^2 and $11y^2$, $2x^3$ and $8x^3$

Note:
Exercise:

Problem: Identify the like terms: $4x^3$, $8x^2$, 19, $3x^2$, 24, $6x^3$.

Solution:

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2 + 3x + 8$, from [link], the three terms are $2x^2, 3x$, and 8.

Example:
Exercise:

Problem: Identify the terms in each expression.

ⓐ $9x^2 + 7x + 12$
ⓑ $8x + 3y$

Solution:
Solution

ⓐ The terms of $9x^2 + 7x + 12$ are $9x^2$, 7*x*, and 12.

ⓑ The terms of $8x + 3y$ are 8*x* and 3*y*.

Note:
Exercise:

Problem: Identify the terms in the expression $4x^2 + 5x + 17$.

Solution:

$4x^2, 5x, 17$

Note:
Exercise:

Problem: Identify the terms in the expression $5x + 2y$.

Solution:

5*x*, 2*y*

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought 12*x*, you would be right!
Equation:

$$\begin{gathered}4x + 7x + x \\ x + x + x + x \quad + x + x + x + x + x + x + x \quad + x \\ 12x\end{gathered}$$

Add the coefficients and keep the same variable. It doesn’t matter what *x* is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

Simplify: $4x + 7x + x$.

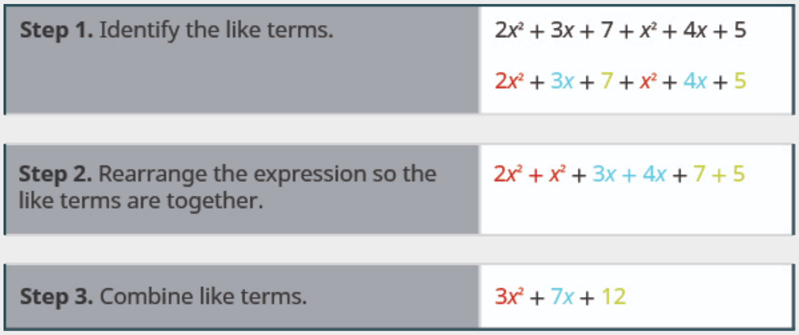
Add the coefficients. 12*x*

Example:

How To Combine Like Terms
Exercise:

Problem: Simplify: $2x^2 + 3x + 7 + x^2 + 4x + 5$.

Solution:
Solution



Step 1. Identify the like terms.	$2x^2 + 3x + 7 + x^2 + 4x + 5$ $2x^2 + 3x + 7 + x^2 + 4x + 5$
Step 2. Rearrange the expression so the like terms are together.	$2x^2 + x^2 + 3x + 4x + 7 + 5$
Step 3. Combine like terms.	$3x^2 + 7x + 12$

Note:
Exercise:

Problem: Simplify: $3x^2 + 7x + 9 + 7x^2 + 9x + 8$.

Solution:

$10x^2 + 16x + 17$

Note:
Exercise:

Problem: Simplify: $4y^2 + 5y + 2 + 8y^2 + 4y + 5$.

Solution:

$12y^2 + 9y + 7$

Note:
Combine Like Terms.

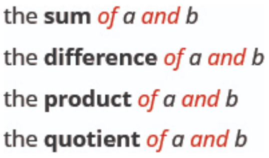
Identify like terms.
Rearrange the expression so like terms are together.
Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra, then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. [link] summarizes them.

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a-b$
Multiplication	a times b the product of a and b twice a	$a\cdot b, ab, a(b), (a)(b)$ $2a$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$

Look closely at these phrases using the four operations:


the **sum** *of a and b*
the **difference** *of a and b*
the **product** *of a and b*
the **quotient** *of a and b*

Each phrase tells us to operate on two numbers. Look for the words *of* and *and* to find the numbers.

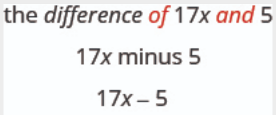
Example:
Exercise:

Problem:

Translate each English phrase into an algebraic expression: ⓐ the difference of $17x$ and 5 ⓑ the quotient of $10x^2$ and 7.

Solution:
Solution

ⓐ The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

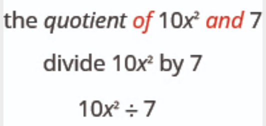


the *difference of* 17*x and* 5

17*x* minus 5

17*x* – 5

ⓑ The key word is “quotient,” which tells us the operation is division.



the *quotient of* $10x^2$ *and* 7

divide $10x^2$ by 7

$10x^2 \div 7$

This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ the difference of $14x^2$ and 13 ⓑ the quotient of 12*x* and 2.

Solution:

ⓐ $14x^2 - 13$ ⓑ $12x \div 2$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ the sum of $17y^2$ and 19 ⓑ the product of 7 and y.

Solution:

ⓐ $17y^2 + 19$ ⓑ $7y$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight “more than” means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven “less than” means 7 subtracted from your present age.

Example:

Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ Seventeen more than *y* ⓑ Nine less than $9x^2$.

Solution:
Solution

ⓐ The key words are *more than*. They tell us the operation is addition. *More than* means "added to."
Equation:

$$\begin{gathered}\text{Seventeen more than } y \\ \text{Seventeen added to } y \\ y+17\end{gathered}$$

ⓑ The key words are *less than*. They tell us to subtract. *Less than* means "subtracted from."
Equation:

$$\begin{gathered}\text{Nine less than } 9x^2 \\ \text{Nine subtracted from } 9x^2 \\ 9x^2-9\end{gathered}$$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ Eleven more than x ⓑ Fourteen less than $11a$.

Solution:

ⓐ $x+11$ ⓑ $11a-14$

Note:
Exercise:

Problem: Translate the English phrase into an algebraic expression: ⓐ 13 more than *z* ⓑ 18 less than 8*x*.

Solution:

ⓐ $z+13$ ⓑ $8x-18$

Example:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ five times the sum of *m* and *n* ⓑ the sum of five times *m* and *n*.

Solution:
Solution

There are two operation words—*times* tells us to multiply and *sum* tells us to add.

ⓐ Because we are multiplying 5 times the sum we need parentheses around the sum of *m* and *n*, $(m+n)$. This forces us to determine the sum first. (Remember the order of operations.)

$$\begin{array}{c}\text{five times the sum of } m \text{ and } n\\ 5\left(m+n\right)\end{array}$$

ⓑ To take a sum, we look for the words “of” and “and” to see what is being added. Here we are taking the sum *of* five times *m* and *n*.

$$\begin{array}{c}\text{the sum of five times } m \text{ and } n\\ 5m+n\end{array}$$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ four times the sum of *p* and *q* ⓑ the sum of four times *p* and *q*.

Solution:

ⓐ $4\left(p+q\right)$ ⓑ $4p+q$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ the difference of two times *x* and 8, ⓑ two times the difference of *x* and 8.

Solution:

ⓐ $2x-8$ ⓑ $2\left(x-8\right)$

Later in this course, we’ll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We’ll see how to do this in the next two examples.

Example:
Exercise:

Problem:

The length of a rectangle is 6 less than the width. Let *w* represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:
Solution

Write a phrase about the length of the rectangle.	6 less than the width
Substitute w for “the width.”	6 less than w
Rewrite “less than” as “subtracted from.”	6 subtracted from w
Translate the phrase into algebra.	$w-6$

Note:
Exercise:

Problem:

The length of a rectangle is 7 less than the width. Let *w* represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:

$w-7$

Note:
Exercise:

Problem:

The width of a rectangle is 6 less than the length. Let *l* represent the length of the rectangle. Write an expression for the width of the rectangle.

Solution:

$l-6$

Example:
Exercise:

Problem:

June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let *q* represent the number of quarters. Write an expression for the number of dimes.

Solution:
Solution

Write the phrase about the number of dimes.	three less than four times the number of quarte
Substitute q for the number of quarters.	3 less than 4 times q
Translate "4 times q."	3 less than $4q$
Translate the phrase into algebra.	$4q - 3$

Note:
Exercise:

Problem:

Geoffrey has dimes and quarters in his pocket. The number of dimes is eight less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution:

$4q - 8$

Note:
Exercise:

Problem:

Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Solution:

$7n + 3$

Key Concepts

- **Notation**

Notation	**The result is…**
$a + b$	the sum of a and b
$a - b$	the difference of a and b
$a \cdot b, ab, (a)(b)\ (a)b, a(b)$	the product of a and b
$a \div b, a/b, \frac{a}{b}, b\overline{)a}$	the quotient of a and b

- **Inequality**

$a < b$ is read "a is less than b"	a is to the left of b on the number line
$a > b$ is read "a is greater than b"	a is to the right of b on the number line

Inequality Symbols	**Words**
$a \neq b$	a is **not equal to** b
$a < b$	a is **less than** b
$a \leq b$	a is **less than or equal to** b
$a > b$	a is **greater than** b
$a \geq b$	a is **greater than or equal to** b

- **Grouping Symbols**
  - Parentheses ( )
  - Brackets [ ]
  - Braces { }
- **Exponential Notation**
  - a^n means multiply a by itself, n times. The expression a^n is read a to the n^{th} power.
- **Order of Operations:** When simplifying mathematical expressions perform the operations in the following order:

  Parentheses and other Grouping Symbols: Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
  Exponents: Simplify all expressions with exponents.
  Multiplication and Division: Perform all multiplication and division in order from left to right. These operations have equal priority.
  Addition and Subtraction: Perform all addition and subtraction in order from left to right. These operations have equal priority.
- **Combine Like Terms**

  Identify like terms.
  Rearrange the expression so like terms are together.
  Add or subtract the coefficients and keep the same variable for each group of like terms.

Practice Makes Perfect

Use Variables and Algebraic Symbols

In the following exercises, translate from algebra to English.

Exercise:

Problem: $16 - 9$

Solution:

16 minus 9, the difference of sixteen and nine

Exercise:

Problem: $3 \cdot 9$

Exercise:

Problem: $28 \div 4$

Solution:

28 divided by 4, the quotient of twenty-eight and four

Exercise:

Problem: $x + 11$

Exercise:

Problem: $(2)\,(7)$

Solution:

2 times 7, the product of two and seven

Exercise:

Problem: $(4)\,(8)$

Exercise:

Problem: $14 < 21$

Solution:

fourteen is less than twenty-one

Exercise:

Problem: $17 < 35$

Exercise:

Problem: $36 \geq 19$

Solution:

thirty-six is greater than or equal to nineteen

Exercise:

Problem: $6n = 36$

Exercise:

Problem: $y - 1 > 6$

Solution:

y minus 1 is greater than 6, the difference of *y* and one is greater than six

Exercise:

Problem: $y - 4 > 8$

Exercise:

Problem: $2 \leq 18 \div 6$

Solution:

2 is less than or equal to 18 divided by 6; 2 is less than or equal to the quotient of eighteen and six

Exercise:

Problem: $a \neq 1 \cdot 12$

In the following exercises, determine if each is an expression or an equation.

Exercise:

Problem: $9 \cdot 6 = 54$

Solution:

equation

Exercise:

Problem: $7 \cdot 9 = 63$

Exercise:

Problem: $5 \cdot 4 + 3$

Solution:

expression

Exercise:

Problem: $x + 7$

Exercise:

Problem: $x + 9$

Solution:

expression

Exercise:

Problem: $y - 5 = 25$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

Exercise:

Problem: 5^3

Solution:

125

Exercise:

Problem: 8^3

Exercise:

Problem: 2^8

Solution:

256

Exercise:

Problem: 10^5

In the following exercises, simplify using the order of operations.

Exercise:

Problem: ⓐ $3+8\cdot 5$ ⓑ $(3+8)\cdot 5$

Solution:

ⓐ 43 ⓑ 55

Exercise:

Problem: ⓐ $2+6\cdot 3$ ⓑ $(2+6)\cdot 3$

Exercise:

Problem: $2^3-12\div(9-5)$

Solution:

5

Exercise:

Problem: $3^2-18\div(11-5)$

Exercise:

Problem: $3\cdot 8+5\cdot 2$

Solution:

34

Exercise:

Problem: $4\cdot 7+3\cdot 5$

Exercise:

Problem: $2+8(6+1)$

Solution:

58

Exercise:

Problem: $4 + 6(3 + 6)$

Exercise:

Problem: $4 \cdot 12/8$

Solution:

6

Exercise:

Problem: $2 \cdot 36/6$

Exercise:

Problem: $(6 + 10) \div (2 + 2)$

Solution:

4

Exercise:

Problem: $(9 + 12) \div (3 + 4)$

Exercise:

Problem: $20 \div 4 + 6 \cdot 5$

Solution:

35

Exercise:

Problem: $33 \div 3 + 8 \cdot 2$

Exercise:

Problem: $3^2 + 7^2$

Solution:

58

Exercise:

Problem: $(3 + 7)^2$

Exercise:

Problem: $3(1 + 9 \cdot 6) - 4^2$

Solution:

149

Exercise:

Problem: $5(2 + 8 \cdot 4) - 7^2$

Exercise:

Problem: $2\left[1 + 3\left(10 - 2\right)\right]$

Solution:

50

Exercise:

Problem: $5\left[2 + 4\left(3 - 2\right)\right]$

Evaluate an Expression

In the following exercises, evaluate the following expressions.

Exercise:

Problem: $7x + 8$ when $x = 2$

Solution:

22

Exercise:

Problem: $8x - 6$ when $x = 7$

Exercise:

Problem: x^2 when $x = 12$

Solution:

144

Exercise:

Problem: x^3 when $x = 5$

Exercise:

Problem: x^5 when $x = 2$

Solution:

32

Exercise:

Problem: 4^x when $x = 2$

Exercise:

Problem: $x^2 + 3x - 7$ when $x = 4$

Solution:

21

Exercise:

Problem: $6x + 3y - 9$ when $x = 6, y = 9$

Exercise:

Problem: $(x - y)^2$ when $x = 10, y = 7$

Solution:

9

Exercise:

Problem: $(x + y)^2$ when $x = 6, y = 9$

Exercise:

Problem: $a^2 + b^2$ when $a = 3, b = 8$

Solution:

73

Exercise:

Problem: $r^2 - s^2$ when $r = 12, s = 5$

Exercise:

Problem: $2l + 2w$ when $l = 15, w = 12$

Solution:

54

Exercise:

Problem: $2l + 2w$ when $l = 18, w = 14$

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.

Exercise:

Problem: $8a$

Solution:

8

Exercise:

Problem: $13m$

Exercise:

Problem: $5r^2$

Solution:

5

Exercise:

Problem: $6x^3$

In the following exercises, identify the like terms.

Exercise:

Problem: $x^3, 8x, 14, 8y, 5, 8x^3$

Solution:

x^3 and $8x^3$, 14 and 5

Exercise:

Problem: $6z, 3w^2, 1, 6z^2, 4z, w^2$

Exercise:

Problem: $9a, a^2, 16, 16b^2, 4, 9b^2$

Solution:

16 and 4, $16b^2$ and $9b^2$

Exercise:

Problem: $3, 25r^2, 10s, 10r, 4r^2, 3s$

In the following exercises, identify the terms in each expression.

Exercise:

Problem: $15x^2 + 6x + 2$

Solution:

$15x^2, 6x, 2$

Exercise:

Problem: $11x^2 + 8x + 5$

Exercise:

Problem: $10y^3 + y + 2$

Solution:

$10y^3, y, 2$

Exercise:

Problem: $9y^3 + y + 5$

In the following exercises, simplify the following expressions by combining like terms.

Exercise:

Problem: $10x + 3x$

Solution:

$13x$

Exercise:

Problem: $15x + 4x$

Exercise:

Problem: $4c + 2c + c$

Solution:

$7c$

Exercise:

Problem: $6y + 4y + y$

Exercise:

Problem: $7u + 2 + 3u + 1$

Solution:

$10u + 3$

Exercise:

Problem: $8d + 6 + 2d + 5$

Exercise:

Problem: $10a + 7 + 5a - 2 + 7a - 4$

Solution:

$22a + 1$

Exercise:

Problem: $7c + 4 + 6c - 3 + 9c - 1$

Exercise:

Problem: $3x^2 + 12x + 11 + 14x^2 + 8x + 5$

Solution:

$17x^2 + 20x + 16$

Exercise:

Problem: $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the phrases into algebraic expressions.

Exercise:

Problem: the difference of 14 and 9

Solution:

$14 - 9$

Exercise:

Problem: the difference of 19 and 8

Exercise:

Problem: the product of 9 and 7

Solution:

$9 \cdot 7$

Exercise:

Problem: the product of 8 and 7

Exercise:

Problem: the quotient of 36 and 9

Solution:

$36 \div 9$

Exercise:

Problem: the quotient of 42 and 7

Exercise:

Problem: the sum of *8x* and 3*x*

Solution:

$8x + 3x$

Exercise:

Problem: the sum of *13x* and 3*x*

Exercise:

Problem: the quotient of *y* and 3

Solution:

$\frac{y}{3}$

Exercise:

Problem: the quotient of *y* and 8

Exercise:

Problem: eight times the difference of *y* and nine

Solution:

$8\left(y - 9\right)$

Exercise:

Problem: seven times the difference of *y* and one

Exercise:

Problem:

Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let *c* represent the number of classical CDs. Write an expression for the number of rock CDs.

Solution:

$c - 4$

Exercise:

Problem:

The number of girls in a second-grade class is 4 less than the number of boys. Let *b* represent the number of boys. Write an expression for the number of girls.

Exercise:

Problem:

Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.

Solution:

$2n-7$

Exercise:

Problem:

Jeannette has $5 and $10 bills in her wallet. The number of fives is three more than six times the number of tens. Let *t* represent the number of tens. Write an expression for the number of fives.

Everyday Math

Exercise:

Problem:

Car insurance Justin's car insurance has a $750 deductible per incident. This means that he pays $750 and his insurance company will pay all costs beyond $750. If Justin files a claim for $2,100.

ⓐ how much will he pay?
ⓑ how much will his insurance company pay?

Solution:

ⓐ $750 ⓑ $1,350

Exercise:

Problem:

Home insurance Armando's home insurance has a $2,500 deductible per incident. This means that he pays $2,500 and the insurance company will pay all costs beyond $2,500. If Armando files a claim for $19,400.

ⓐ how much will he pay?
ⓑ how much will the insurance company pay?

Writing Exercises

Exercise:

Problem: Explain the difference between an expression and an equation.

Solution:

Answers may vary

Exercise:

Problem: Why is it important to use the order of operations to simplify an expression?

Exercise:

Problem: Explain how you identify the like terms in the expression $8a^2+4a+9-a^2-1$.

Solution:

Answers may vary

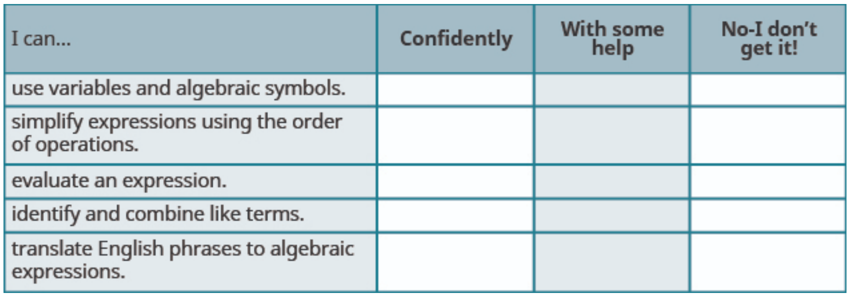
Exercise:

Problem:

Explain the difference between the phrases “4 times the sum of x and y” and “the sum of 4 times x and y.”

Self Check

ⓐ Use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
use variables and algebraic symbols.			
simplify expressions using the order of operations.			
evaluate an expression.			
identify and combine like terms.			
translate English phrases to algebraic expressions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

coefficient
The coefficient of a term is the constant that multiplies the variable in a term.

constant
A constant is a number whose value always stays the same.

equality symbol
The symbol “$=$” is called the equal sign. We read $a = b$ as “a is equal to b.”

equation
An equation is two expressions connected by an equal sign.

evaluate an expression
To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

expression
An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

like terms
Terms that are either constants or have the same variables raised to the same powers are called like terms.

simplify an expression
To simplify an expression, do all operations in the expression.

term
A term is a constant or the product of a constant and one or more variables.

variable

A variable is a letter that represents a number whose value may change.

Basic Operations with Real Numbers: Multiplication and Division of Signed Numbers
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The basic operations with real numbers are presented in this chapter. The concept of absolute value is discussed both geometrically and symbolically. The geometric presentation offers a visual understanding of the meaning of |x|. The symbolic presentation includes a literal explanation of how to use the definition. Negative exponents are developed, using reciprocals and the rules of exponents the student has already learned. Scientific notation is also included, using unique and real-life examples. Objectives of this module: be able to multiply and divide signed numbers.

Overview

- Multiplication of Signed Numbers
- Division of Signed Numbers

Multiplication of Signed Numbers

Let us consider first the product of two positive numbers.

Multiply: $3 \cdot 5$.
$3 \cdot 5$ means $5 + 5 + 5 = 15$.

This suggests that

$(\text{positive number}) \cdot (\text{positive number}) = \text{positive number}.$

More briefly, $(+)(+) = +$.

Now consider the product of a positive number and a negative number.

Multiply: $(3)(-5)$.
$(3)(-5)$ means $(-5) + (-5) + (-5) = -15$.

This suggests that

$(\text{positive number}) \cdot (\text{negative number}) = \text{negative number}$

More briefly, $(+)(-) = -$.

By the commutative property of multiplication, we get

$(\text{negative number}) \cdot (\text{positive number}) = \text{negative number}$

More briefly, $(-)(+) = -$.

The sign of the product of two negative numbers can be determined using the following illustration: Multiply -2 by, respectively, $4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ -3,\ -4$. Notice that when the multiplier decreases by 1, the product increases by 2.

$$\left.\begin{array}{l} 4(-2) = -8 \\ 3(-2) = -6 \\ 2(-2) = -4 \\ 1(-2) = -2 \end{array}\right\} \rightarrow \text{As we know, } (+)(-) = -.$$

$$0(-2) = 0 \quad \rightarrow \quad \text{As we know, } 0 \cdot (\text{any number}) = 0.$$

$$\left.\begin{array}{l} -1(-2) = 2 \\ -2(-2) = 4 \\ -3(-2) = 6 \\ -4(-2) = 8 \end{array}\right\} \rightarrow \text{This pattern suggests } (-)(-) = +.$$

We have the following rules for multiplying signed numbers.

Rules for Multiplying Signed Numbers
To multiply two real numbers that have

1. the **same sign**, multiply their absolute values. The product is positive.
   $(+)(+) = +$
   $(-)(-) = +$
2. **opposite signs**, multiply their absolute values. The product is negative.
   $(+)(-) = -$
   $(-)(+) = -$

Sample Set A

Find the following products.

Example:
$8 \cdot 6$

Multiply these absolute values.

$$\left.\begin{array}{l}|8| = 8\\ |6| = 6\end{array}\right\} \quad 8 \cdot 6 = 48$$

Since the numbers have the same sign, the product is positive.

$8 \cdot 6 = +48$ or $8 \cdot 6 = 48$

Example:
$(-8)(-6)$

Multiply these absolute values.

$$\left.\begin{array}{l}|-8| = 8\\ |-6| = 6\end{array}\right\} \quad 8 \cdot 6 = 48$$

Since the numbers have the same sign, the product is positive.

$(-8)(-6) = +48$ or $(-8)(-6) = 48$

Example:
$(-4)(7)$

Multiply these absolute values.

$$\left.\begin{array}{ll}|-4| & = 4\\ |7| & = 7\end{array}\right\} \quad 4 \cdot 7 = 28$$

Since the numbers have opposite signs, the product is negative.

$(-4)(7) = -28$

Example:
$6(-3)$

Multiply these absolute values.

$$\left.\begin{array}{ll}|6| & = 6\\ |-3| & = 3\end{array}\right\} \quad 6 \cdot 3 = 18$$

Since the numbers have opposite signs, the product is negative.

$6(-3) = -18$

Practice Set A

Find the following products.

Exercise:

Problem: $3(-8)$

Solution:

-24

Exercise:

Problem: $4(16)$

Solution:

64

Exercise:

Problem: $(-6)(-5)$

Solution:

30

Exercise:

Problem: $(-7)(-2)$

Solution:

14

Exercise:

Problem: $(-1)(4)$

Solution:

-4

Exercise:

Problem: $(-7)7$

Solution:

-49

Division of Signed Numbers

We can determine the sign pattern for division by relating division to multiplication. Division is defined in terms of multiplication in the following way.

If $b \cdot c = a$, then $\frac{a}{b} = c$, $b \neq 0$.

For example, since $3 \cdot 4 = 12$, it follows that $\frac{12}{3} = 4$.

Notice the pattern:

Since $\underset{b\cdot c=a}{3 \cdot 4 = 12}$, it follows that $\underset{\frac{a}{b}=c}{\frac{12}{3} = 4}$

The sign pattern for division follows from the sign pattern for multiplication.

1. Since $\underset{b\cdot c=a}{(+)(+) = +}$, it follows that $\underset{\frac{a}{b}=c}{\frac{(+)}{(+)} = +}$, that is,

   $$\frac{(\text{positive number})}{(\text{positive number})} = \text{positive number}$$

2. Since $\underset{b\cdot c=a}{(-)(-) = +}$, it follows that $\underset{\frac{a}{b}=c}{\frac{(+)}{(-)} = -}$, that is,

   $$\frac{(\text{positive number})}{(\text{negative number})} = \text{negative number}$$

3. Since $\underset{b\cdot c=a}{(+)(-) = -}$, it follows that $\underset{\frac{a}{b}=c}{\frac{(-)}{(+)} = -}$, that is,

   $$\frac{(\text{negative number})}{(\text{positive number})} = \text{negative number}$$

4. Since $\underset{b\cdot c=a}{(-)(+) = -}$, it follows that $\underset{\frac{a}{b}=c}{\frac{(-)}{(-)} = +}$, that is

$$\frac{(\text{negative number})}{(\text{negative number})} = \text{positive number}$$

We have the following rules for dividing signed numbers.

Rules for Dividing Signed Numbers
To divide two real numbers that have

1. the **same sign**, divide their absolute values. The quotient is positive.
   $\frac{(+)}{(+)} = + \quad \frac{(-)}{(-)} = +$
2. **opposite signs**, divide their absolute values. The quotient is negative.
   $\frac{(-)}{(+)} = - \quad \frac{(+)}{(-)} = -$

Sample Set B

Find the following quotients.

Example:

$\frac{-10}{2}$

$$\left.\begin{aligned} |-10| &= 10 \\ |2| &= 2 \end{aligned}\right\} \text{Divide these absolute values.}$$

$$\frac{10}{2} = 5$$

$$\frac{-10}{2} = -5 \quad \text{Since the numbers have opposite signs, the quotient is negative.}$$

Example:

$\frac{-35}{-7}$

$$\left.\begin{aligned} |-35| &= 35 \\ |-7| &= 7 \end{aligned}\right\} \text{Divide these absolute values.}$$

$$\frac{35}{7} = 5$$

$$\frac{-35}{-7} = 5 \quad \text{Since the numbers have same signs, the quotient is positive.}$$

Example:

$\frac{18}{-9}$

$$\begin{array}{lll} \left.\begin{array}{ll} |18| = & 18 \\ |-9| = & 9 \end{array}\right\} & & \text{Divide these absolute values.} \\ & & \frac{18}{9} = 2 \\ \frac{18}{-9} & = -2 & \text{Since the numbers have opposite signs, the quotient is negative.} \end{array}$$

Practice Set B

Find the following quotients.

Exercise:

Problem: $\frac{-24}{-6}$

Solution:

4

Exercise:

Problem: $\frac{30}{-5}$

Solution:

-6

Exercise:

Problem: $\frac{-54}{27}$

Solution:

-2

Exercise:

Problem: $\frac{51}{17}$

Solution:

3

Sample Set C

Example:
Find the value of $\frac{-6(4-7)-2(8-9)}{-(4+1)+1}$.
Using the order of operations and what we know about signed numbers, we get

$$\begin{aligned}\frac{-6(4-7)-2(8-9)}{-(4+1)+1} &= \frac{-6(-3)-2(-1)}{-(5)+1}\\ &= \frac{18+2}{-5+1}\\ &= \frac{20}{-4}\\ &= -5\end{aligned}$$

Example:
Find the value of $z = \frac{x-u}{s}$ if $x = 57$, $u = 51$, and $s = 2$.
Substituting these values we get
$z = \frac{57-51}{2} = \frac{6}{2} = 3$

Practice Set C

Exercise:

Problem: Find the value of $\frac{-7(4-8)+2(1-11)}{-5(1-6)-17}$.

Solution:

1

Exercise:

Problem: Find the value of $P = \frac{n(n-3)}{2n}$, if $n = 5$.

Solution:

1

Exercises

Find the value of each of the following expressions.

Exercise:

Problem: $(-2)(-8)$

Solution:

16

Exercise:

Problem: $(-3)(-9)$

Exercise:

Problem: $(-4)(-8)$

Solution:

32

Exercise:

Problem: $(-5)(-2)$

Exercise:

Problem: $(-6)(-9)$

Solution:

54

Exercise:

Problem: $(-3)(-11)$

Exercise:

Problem: $(-8)(-4)$

Solution:

32

Exercise:

Problem: $(-1)(-6)$

Exercise:

Problem: $(3)(-12)$

Solution:

-36

Exercise:

Problem: $(4)(-18)$

Exercise:

Problem: $8(-4)$

Solution:

-32

Exercise:

Problem: $5(-6)$

Exercise:

Problem: $9(-2)$

Solution:

-18

Exercise:

Problem: $7(-8)$

Exercise:

Problem: $(-6)4$

Solution:

-24

Exercise:

Problem: $(-7)6$

Exercise:

Problem: $(-10)9$

Solution:

-90

Exercise:

Problem: $(-4)12$

Exercise:

Problem: $(10)(-6)$

Solution:

-60

Exercise:

Problem: $(-6)(4)$

Exercise:

Problem: $(-2)(6)$

Solution:

-12

Exercise:

Problem: $(-8)(7)$

Exercise:

Problem: $\frac{21}{7}$

Solution:

3

Exercise:

Problem: $\frac{42}{6}$

Exercise:

Problem: $\frac{-39}{3}$

Solution:

-13

Exercise:

Problem: $\frac{-20}{10}$

Exercise:

Problem: $\frac{-45}{-5}$

Solution:

9

Exercise:

Problem: $\frac{-16}{-8}$

Exercise:

Problem: $\frac{25}{-5}$

Solution:

-5

Exercise:

Problem: $\frac{36}{-4}$

Exercise:

Problem: $8-(-3)$

Solution:

11

Exercise:

Problem: $14-(-20)$

Exercise:

Problem: $20-(-8)$

Solution:

28

Exercise:

Problem: $-4-(-1)$

Exercise:

Problem: $0-4$

Solution:

-4

Exercise:

Problem: $0-(-1)$

Exercise:

Problem: $-6 + 1 - 7$

Solution:

-12

Exercise:

Problem: $15 - 12 - 20$

Exercise:

Problem: $1 - 6 - 7 + 8$

Solution:

-4

Exercise:

Problem: $2 + 7 - 10 + 2$

Exercise:

Problem: $3(4 - 6)$

Solution:

-6

Exercise:

Problem: $8(5 - 12)$

Exercise:

Problem: $-3(1 - 6)$

Solution:

15

Exercise:

Problem: $-8(4-12)+2$

Exercise:

Problem: $-4(1-8)+3(10-3)$

Solution:

49

Exercise:

Problem: $-9(0-2)+4(8-9)+0(-3)$

Exercise:

Problem: $6(-2-9)-6(2+9)+4(-1-1)$

Solution:

-140

Exercise:

Problem: $\frac{3(4+1)-2(5)}{-2}$

Exercise:

Problem: $\frac{4(8+1)-3(-2)}{-4-2}$

Solution:

-7

Exercise:

Problem: $\frac{-1(3+2)+5}{-1}$

Exercise:

Problem: $\frac{-3(4-2)+(-3)(-6)}{-4}$

Solution:

-3

Exercise:

Problem: $-1(4+2)$

Exercise:

Problem: $-1(6-1)$

Solution:

-5

Exercise:

Problem: $-(8+21)$

Exercise:

Problem: $-(8-21)$

Solution:

13

Exercise:

Problem: $-(10-6)$

Exercise:

Problem: $-(5-2)$

Solution:

-3

Exercise:

Problem: $-(7-11)$

Exercise:

Problem: $-(8-12)$

Solution:

4

Exercise:

Problem: $-3[(-1+6)-(2-7)]$

Exercise:

Problem: $-2[(4-8)-(5-11)]$

Solution:

-4

Exercise:

Problem: $-5[(-1+5)+(6-8)]$

Exercise:

Problem: $-[(4-9)+(-2-8)]$

Solution:

15

Exercise:

Problem: $-3[-2(1-5)-3(-2+6)]$

Exercise:

Problem: $-2[-5(-10+11)-2(5-7)]$

Solution:

2

Exercise:

Problem: $P = R - C.\quad$ Find P if $R = 2000$ and $C = 2500.$

Exercise:

Problem: $z = \frac{x-u}{s}.\quad$ Find z if $x = 23,\ u = 25,$ and $s = 1.$

Solution:

-2

Exercise:

Problem: $z = \frac{x-u}{s}.\quad$ Find z if $x = 410,\ u = 430,$ and $s = 2.5.$

Exercise:

Problem: $m = \frac{2s+1}{T}.\quad$ Find m if $s = -8$ and $T = 5.$

Solution:

-3

Exercise:

Problem: $m = \frac{2s+1}{T}.\quad$ Find m if $s = -10$ and $T = -5.$

Exercise:

Problem: $F = (p_1 - p_2)r^4 \cdot 9.\quad$ Find F if $p_1 = 10,\ p_2 = 8, r = 3.$

Solution:

1458

Exercise:

Problem: $F = (p_1 - p_2)r^4 \cdot 9.\quad \text{Find } F \text{ if } p_1 = 12,\ p_2 = 7, r = 2.$

Exercise:

Problem: $P = n(n-1)(n-2).\quad \text{Find } P \text{ if } n = -4.$

Solution:

-120

Exercise:

Problem: $P = n(n-1)(n-2)(n-3).\quad \text{Find } P \text{ if } n = -5.$

Exercise:

Problem: $P = \frac{n(n-2)(n-4)}{2n}.\quad \text{Find } P \text{ if } n = -6.$

Solution:

40

Exercises for Review

Exercise:

Problem:

([link]) What natural numbers can replace x so that the statement $-4 < x \leq 3$ is true?

Exercise:

Problem: ([link]) Simplify $\frac{(x+2y)^5(3x-1)^7}{(x+2y)^3(3x-1)^6}$.

Solution:

$(x+2y)^2(3x-1)$

Exercise:

Problem: (**[link]**) Simplify $\left(x^n y^{3t}\right)^5$.

Exercise:

Problem: (**[link]**) Find the sum. $-6 + (-5)$.

Solution:

-11

Exercise:

Problem: (**[link]**) Find the difference. $-2 - (-8)$.

Understand Percent

By the end of this section, you will be able to:

- Use the definition of percent
- Convert percents to fractions and decimals
- Convert decimals and fractions to percents

Note:
Before you get started, take this readiness quiz.

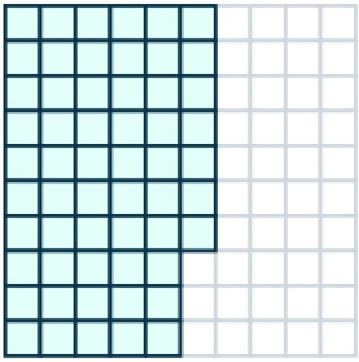
1. Translate “the ratio of 33 to 5” into an algebraic expression.
   If you missed this problem, review [link].
2. Write $\frac{3}{5}$ as a decimal.
   If you missed this problem, review [link].
3. Write 0.62 as a fraction.
   If you missed this problem, review [link].

Use the Definition of Percent

How many cents are in one dollar? There are 100 cents in a dollar. How many years are in a century? There are 100 years in a century. Does this give you a clue about what the word “percent” means? It is really two words, “per cent,” and means per one hundred. A **percent** is a ratio whose denominator is 100. We use the percent symbol $\%$, to show percent.

Note:
Percent
A percent is a ratio whose denominator is 100.

According to data from the American Association of Community Colleges (2015), about 57% of community college students are female. This means 57 out of every 100 community college students are female, as [link] shows. Out of the 100 squares on the grid, 57 are shaded, which we write as the ratio $\frac{57}{100}$.



Among every 100 community college students, 57 are female.

Similarly, 25% means a ratio of $\frac{25}{100}$, 3% means a ratio of $\frac{3}{100}$ and 100% means a ratio of $\frac{100}{100}$. In words, "one hundred percent" means the total 100% is $\frac{100}{100}$, and since $\frac{100}{100} = 1$, we see that 100% means 1 whole.

Example:
Exercise:

Problem:

According to the Public Policy Institute of California (2010), 44% of parents of public school children would like their youngest child to earn a graduate degree. Write this percent as a ratio.

Solution:
Solution

The amount we want to convert is 44%.	44%
Write the percent as a ratio. Remember that *percent* means per 100.	$\frac{44}{100}$

Note:
Exercise:

Problem: Write the percent as a ratio.

According to a survey, 89% of college students have a smartphone.

Solution:

$\frac{89}{100}$

Note:
Exercise:

Problem: Write the percent as a ratio.

A study found that 72% of U.S. teens send text messages regularly.

Solution:

$\frac{72}{100}$

Example:
Exercise:

Problem:

In 2007, according to a U.S. Department of Education report, 21 out of every 100 first-time freshmen college students at 4-year public institutions took at least one remedial course. Write this as a ratio and then as a percent.

Solution:
Solution

The amount we want to convert is 21 out of 100.	21 out of 100
Write as a ratio.	$\frac{21}{100}$
Convert the 21 per 100 to percent.	21%

Note:
Exercise:

Problem:

Write as a ratio and then as a percent: The American Association of Community Colleges reported that 62 out of 100 full-time community college students balance their studies with full-time or part time employment.

Solution:

$\frac{62}{100}, 62\%$

Note:
Exercise:

Problem:

Write as a ratio and then as a percent: In response to a student survey, 41 out of 100 Santa Ana College students expressed a goal of earning an Associate's degree or transferring to a four-year college.

Solution:

$\frac{41}{100}, 41\%$

Convert Percents to Fractions and Decimals

Since percents are ratios, they can easily be expressed as fractions. Remember that percent means per 100, so the denominator of the fraction is 100.

Note:

Convert a percent to a fraction.

Write the percent as a ratio with the denominator 100.
Simplify the fraction if possible.

Example:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 36%
ⓑ 125%

Solution:
Solution

ⓐ	
	36%
Write as a ratio with denominator 100.	$\frac{36}{100}$
Simplify.	$\frac{9}{25}$

ⓑ	
	125%
Write as a ratio with denominator 100.	$\frac{125}{100}$
Simplify.	$\frac{5}{4}$

Note:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 48%
ⓑ 110%

Solution:

ⓐ $\frac{12}{25}$
ⓑ $\frac{11}{10}$

Note:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 64%
ⓑ 150%

Solution:

ⓐ $\frac{16}{25}$
ⓑ $\frac{3}{2}$

The previous example shows that a percent can be greater than 1. We saw that 125% means $\frac{125}{100}$, or $\frac{5}{4}$. These are improper fractions, and their values are greater than one.

Example:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 24.5%
ⓑ $33\frac{1}{3}\%$

Solution:
Solution

ⓐ	
	24.5%

Write as a ratio with denominator 100.	$\frac{24.5}{100}$
Clear the decimal by multiplying numerator and denominator by 10.	$\frac{24.5(10)}{100(10)}$
Multiply.	$\frac{245}{1000}$
Rewrite showing common factors.	$\frac{5\cdot 49}{5\cdot 200}$
Simplify.	$\frac{49}{200}$

ⓑ	
	$33\frac{1}{3}\%$
Write as a ratio with denominator 100.	$\frac{33\frac{1}{3}}{100}$
Write the numerator as an improper fraction.	$\frac{\frac{100}{3}}{100}$
Rewrite as fraction division, replacing 100 with $\frac{100}{1}$.	$\frac{100}{3} \div \frac{100}{1}$
Multiply by the reciprocal.	$\frac{100}{3} \cdot \frac{1}{100}$
Simplify.	$\frac{1}{3}$

Note:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 64.4%
ⓑ $66\frac{2}{3}\%$

Solution:

ⓐ $\frac{161}{250}$
ⓑ $\frac{2}{3}$

Note:
Exercise:

Problem: Convert each percent to a fraction:

ⓐ 42.5%
ⓑ $8\frac{3}{4}\%$

Solution:

ⓐ $\frac{113}{250}$
ⓑ $\frac{7}{80}$

In Decimals, we learned how to convert fractions to decimals. To convert a percent to a decimal, we first convert it to a fraction and then change the fraction to a decimal.

Note:
Convert a percent to a decimal.

Write the percent as a ratio with the denominator 100.
Convert the fraction to a decimal by dividing the numerator by the denominator.

Example:
Exercise:

Problem: Convert each percent to a decimal:

ⓐ 6%
ⓑ 78%

Solution:
Solution

Because we want to change to a decimal, we will leave the fractions with denominator 100 instead of removing common factors.

ⓐ	
	6%

Write as a ratio with denominator 100.	$\frac{6}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06

ⓑ	
	78%
Write as a ratio with denominator 100.	$\frac{78}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.78

Note:
Exercise:

Problem: Convert each percent to a decimal:

ⓐ 9%
ⓑ 87%

Solution:

ⓐ 0.09
ⓑ 0.87

Note:
Exercise:

Problem: Convert each percent to a decimal:

ⓐ 3%
ⓑ 91%

Solution:

ⓐ 0.03
ⓑ 0.91

Example:
Exercise:

Problem: Convert each percent to a decimal:

ⓐ 135%
ⓑ 12.5%

Solution:
Solution

ⓐ	
	135%
Write as a ratio with denominator 100.	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	1.35

ⓑ	
	12.5%
Write as a ratio with denominator 100.	$\frac{12.5}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.125

Note:
Exercise:

Problem: Convert each percent to a decimal:

ⓐ 115%
ⓑ 23.5%

Solution:

ⓐ 1.15
ⓑ 0.235

Note:
Exercise:

Problem: Convert each percent to a decimal:

ⓐ 123%
ⓑ 16.8%

Solution:

ⓐ 1.23
ⓑ 0.168

Let's summarize the results from the previous examples in [link], and look for a pattern we could use to quickly convert a percent number to a decimal number.

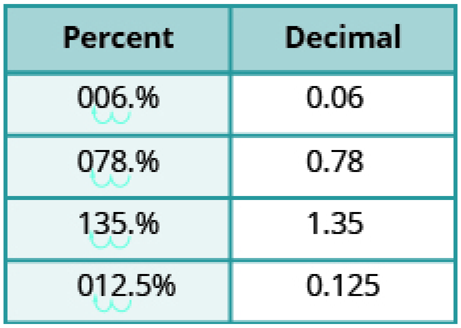
Percent	**Decimal**
6%	0.06
78%	0.78

Percent	**Decimal**
135%	1.35
12.5%	0.125

Do you see the pattern?

To convert a percent number to a decimal number, we move the decimal point two places to the left and remove the % sign. (Sometimes the decimal point does not appear in the percent number, but just like we can think of the integer 6 as 6.0, we can think of 6% as 6.0%.) Notice that we may need to add zeros in front of the number when moving the decimal to the left.

[link] uses the percents in [link] and shows visually how to convert them to decimals by moving the decimal point two places to the left.



Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Example:
Exercise:

Problem:

Among a group of business leaders, 77% believe that poor math and science education in the U.S. will lead to higher unemployment rates.

Convert the percent to: ⓐ a fraction ⓑ a decimal

Solution:
Solution

ⓐ	
	77%
Write as a ratio with denominator 100.	$\frac{77}{100}$

ⓑ	
	$\frac{77}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.77

Note:
Exercise:

Problem: Convert the percent to: ⓐ a fraction and ⓑ a decimal

Twitter's share of web traffic jumped 24% when one celebrity tweeted live on air.

Solution:

ⓐ $\frac{6}{25}$
ⓑ 0.24

Note:
Exercise:

Problem: Convert the percent to: ⓐ a fraction and ⓑ a decimal

The U.S. Census estimated that in 2013, 44% of the population of Boston age 25 or older have a bachelor's or higher degrees.

Solution:

ⓐ $\frac{22}{50}$
ⓑ 0.44

Example:
Exercise:

Problem:

There are four suits of cards in a deck of cards—hearts, diamonds, clubs, and spades. The probability of randomly choosing a heart from a shuffled deck of cards is 25%. Convert the percent to:

ⓐ a fraction

ⓑ a decimal

(credit: Riles32807, Wikimedia Commons)

Solution:
Solution

ⓐ	
	25%
Write as a ratio with denominator 100.	$\frac{25}{100}$
Simplify.	$\frac{1}{4}$

ⓑ	$\frac{1}{4}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.25

Note:
Exercise:

Problem: Convert the percent to: ⓐ a fraction, and ⓑ a decimal

The probability that it will rain Monday is 30%.

Solution:

ⓐ $\frac{3}{10}$
ⓑ 0.3

Note:
Exercise:

Problem: Convert the percent to: ⓐ a fraction, and ⓑ a decimal

The probability of getting heads three times when tossing a coin three times is 12.5%.

Solution:

ⓐ $\frac{12.5}{100}$
ⓑ 0.125

Convert Decimals and Fractions to Percents

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

Note:
Convert a decimal to a percent.

Write the decimal as a fraction.
If the denominator of the fraction is not $100,$ rewrite it as an equivalent fraction with denominator $100.$
Write this ratio as a percent.

Example:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 0.05 ⓑ 0.83

Solution:
Solution

ⓐ	

	0.05
Write as a fraction. The denominator is 100.	$\frac{5}{100}$
Write this ratio as a percent.	5%

ⓑ	
	0.83
The denominator is 100.	$\frac{83}{100}$
Write this ratio as a percent.	83%

Note:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 0.01 ⓑ 0.17.

Solution:

ⓐ 1%
ⓑ 17%

Note:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 0.04 ⓑ 0.41

Solution:

ⓐ 4%
ⓑ 41%

To convert a mixed number to a percent, we first write it as an improper fraction.

Example:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 1.05 ⓑ 0.075

Solution:
Solution

ⓐ	
	1.05
Write as a fraction.	$1\frac{5}{100}$

Write as an improper fraction. The denominator is 100.	$\frac{105}{100}$
Write this ratio as a percent.	105%

Notice that since $1.05 > 1$, the result is more than 100%.

ⓑ	
	0.075
Write as a fraction. The denominator is 1,000.	$\frac{75}{1,000}$
Divide the numerator and denominator by 10, so that the denominator is 100.	$\frac{7.5}{100}$
Write this ratio as a percent.	7.5%

Note:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 1.75 ⓑ 0.0825

Solution:

ⓐ 175%
ⓑ 8.25%

Note:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 2.25 ⓑ 0.0925

Solution:

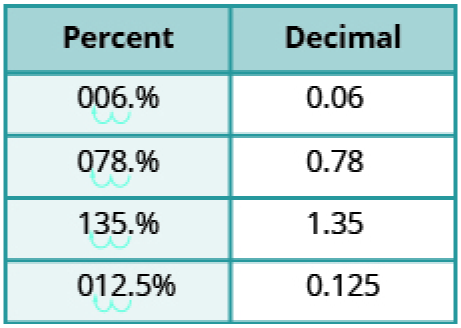
ⓐ 225%
ⓑ 9.25%

Let's summarize the results from the previous examples in [link] so we can look for a pattern.

Decimal	Percent
0.05	5%
0.83	83%
1.05	105%
0.075	7.5%

Do you see the pattern? To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.

[link] uses the decimal numbers in [link] and shows visually to convert them to percents by moving the decimal point two places to the right and then writing the % sign.



Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

In Decimals, we learned how to convert fractions to decimals. Now we also know how to change decimals to percents. So to convert a fraction to a percent, we first change it to a decimal and then convert that decimal to a percent.

Note:
Convert a fraction to a percent.

Convert the fraction to a decimal.
Convert the decimal to a percent.

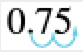
Example:
Exercise:

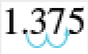
Problem:

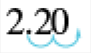
Convert each fraction or mixed number to a percent: ⓐ $\frac{3}{4}$ ⓑ $\frac{11}{8}$ ⓒ $2\frac{1}{5}$

Solution:
Solution

To convert a fraction to a decimal, divide the numerator by the denominator.

ⓐ	
Change to a decimal.	$\frac{3}{4}$
Write as a percent by moving the decimal two places.	 0.75
	75%

ⓑ	
Change to a decimal.	$\frac{11}{8}$
Write as a percent by moving the decimal two places.	 1.375
	137.5%

ⓒ	
Write as an improper fraction.	$2\frac{1}{5}$
Change to a decimal.	$\frac{11}{5}$
Write as a percent.	 2.20
	220%

Notice that we needed to add zeros at the end of the number when moving the decimal two places to the right.

Note:
Exercise:

Problem:

Convert each fraction or mixed number to a percent: ⓐ $\frac{5}{8}$ ⓑ $\frac{11}{4}$ ⓒ $3\frac{2}{5}$

Solution:

ⓐ 62.5%
ⓑ 275%
ⓒ 340%

Note:
Exercise:

Problem:

Convert each fraction or mixed number to a percent: ⓐ $\frac{7}{8}$ ⓑ $\frac{9}{4}$ ⓒ $1\frac{3}{5}$

Solution:

ⓐ 87.5%
ⓑ 225%
ⓒ 160%

Sometimes when changing a fraction to a decimal, the division continues for many decimal places and we will round off the quotient. The number of decimal places we round to will depend on the situation. If the decimal involves money, we round to the hundredths place. For most other cases in this book we will round the number to the nearest thousandth, so the percent will be rounded to the nearest tenth.

Example:
Exercise:

Problem: Convert $\frac{5}{7}$ to a percent.

Solution:
Solution

To change a fraction to a decimal, we divide the numerator by the denominator.

	$\frac{5}{7}$
Change to a decimal—rounding to the nearest thousandth.	0.714
Write as a percent.	71.4%

Note:
Exercise:

Problem: Convert the fraction to a percent: $\frac{3}{7}$

Solution:

42.9%

Note:
Exercise:

Problem: Convert the fraction to a percent: $\frac{4}{7}$

Solution:

57.1%

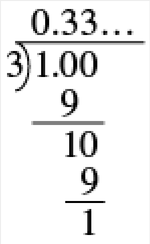
When we first looked at fractions and decimals, we saw that fractions converted to a repeating decimal. When we converted the fraction $\frac{4}{3}$ to a decimal, we wrote the answer as 1.3. We will use this same notation, as well as fraction notation, when we convert fractions to percents in the next example.

Example:
Exercise:

Problem:

An article in a medical journal claimed that approximately $\frac{1}{3}$ of American adults are obese. Convert the fraction $\frac{1}{3}$ to a percent.

Solution:
Solution

	$\frac{1}{3}$
Change to a decimal.	
Write as a repeating decimal.	$0.333\ldots$
Write as a percent.	$33\frac{1}{3}\%$

We could also write the percent as $33.\overline{3}\%$.

Note:
Exercise:

Problem: Convert the fraction to a percent:

According to the U.S. Census Bureau, about $\frac{1}{9}$ of United States housing units have just 1 bedroom.

Solution:

$11.\overline{1}\%$, or $11\frac{1}{9}\%$

Note:
Exercise:

Problem: Convert the fraction to a percent:

According to the U.S. Census Bureau, about $\frac{1}{6}$ of Colorado residents speak a language other than English at home.

Solution:

$16.\overline{6}\%$, or $16\frac{2}{3}\%$

Key Concepts

- **Convert a percent to a fraction.**

Write the percent as a ratio with the denominator 100.
Simplify the fraction if possible.

- **Convert a percent to a decimal.**

  Write the percent as a ratio with the denominator 100.
  Convert the fraction to a decimal by dividing the numerator by the denominator.

- **Convert a decimal to a percent.**

  Write the decimal as a fraction.
  If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.
  Write this ratio as a percent.

- **Convert a fraction to a percent.**

  Convert the fraction to a decimal.
  Convert the decimal to a percent.

Practice Makes Perfect

Use the Definition of Percents

In the following exercises, write each percent as a ratio.

Exercise:

Problem:

In 2014, the unemployment rate for those with only a high school degree was 6.0%.

Solution:

$\frac{6}{100}$

Exercise:

Problem:

In 2015, among the unemployed, 29% were long-term unemployed.

Exercise:

Problem:

The unemployment rate for those with Bachelor's degrees was 3.2% in 2014.

Solution:

$\frac{32}{1000}$

Exercise:

Problem: The unemployment rate in Michigan in 2014 was 7.3%.

In the following exercises, write as

ⓐ a ratio and
ⓑ a percent

Exercise:

Problem:

57 out of 100 nursing candidates received their degree at a community college.

Solution:

ⓐ $\frac{57}{100}$
ⓑ 57%

Exercise:

Problem:

80 out of 100 firefighters and law enforcement officers were educated at a community college.

Exercise:

Problem:

42 out of 100 first-time freshmen students attend a community college.

Solution:

ⓐ $\frac{42}{100}$
ⓑ 42%

Exercise:

Problem:

71 out of 100 full-time community college faculty have a master's degree.

Convert Percents to Fractions and Decimals

In the following exercises, convert each percent to a fraction and simplify all fractions.

Exercise:

Problem: 4%

Solution:

$\frac{1}{25}$

Exercise:

Problem: 8%

Exercise:

Problem: 17%

Solution:

$\frac{17}{100}$

Exercise:

Problem: 19%

Exercise:

Problem: 52%

Solution:

$\frac{13}{25}$

Exercise:

Problem: 78%

Exercise:

Problem: 125%

Solution:

$\frac{5}{4}$

Exercise:

Problem: 135%

Exercise:

Problem: 37.5%

Solution:

$\frac{3}{8}$

Exercise:

Problem: 42.5%

Exercise:

Problem: 18.4%

Solution:

$\frac{23}{125}$

Exercise:

Problem: 46.4%

Exercise:

Problem: $9\frac{1}{2}\%$

Solution:

$\frac{19}{200}$

Exercise:

Problem: $8\frac{1}{2}\%$

Exercise:

Problem: $5\frac{1}{3}\%$

Solution:

$\frac{4}{75}$

Exercise:

Problem: $6\frac{2}{3}\%$

In the following exercises, convert each percent to a decimal.

Exercise:

Problem: 5%

Solution:

0.05

Exercise:

Problem: 9%

Exercise:

Problem: 1%

Solution:

0.01

Exercise:

Problem: 2%

Exercise:

Problem: 63%

Solution:

0.63

Exercise:

Problem: 71%

Exercise:

Problem: 40%

Solution:

0.4

Exercise:

Problem: 50%

Exercise:

Problem: 115%

Solution:

1.15

Exercise:

Problem: 125%

Exercise:

Problem: 150%

Solution:

1.5

Exercise:

Problem: 250%

Exercise:

Problem: 21.4%

Solution:

0.214

Exercise:

Problem: 39.3%

Exercise:

Problem: 7.8%

Solution:

0.078

Exercise:

Problem: 6.4%

In the following exercises, convert each percent to

ⓐ a simplified fraction and
ⓑ a decimal

Exercise:

Problem:

In 2010, 1.5% of home sales had owner financing. (*Source:* Bloomberg Businessweek, 5/23–29/2011)

Solution:

ⓐ $\frac{3}{200}$
ⓑ 0.015

Exercise:

Problem:

In 2000, 4.2% of the United States population was of Asian descent. (*Source:* www.census.gov)

Exercise:

Problem:

According to government data, in 2013 the number of cell phones in India was 70.23% of the population.

Solution:

ⓐ $\frac{7023}{10{,}000}$
ⓑ 0.7023

Exercise:

Problem:

According to the U.S. Census Bureau, among Americans age 25 or older who had doctorate degrees in 2014, 37.1% are women.

Exercise:

Problem:

A couple plans to have two children. The probability they will have two girls is 25%.

Solution:

ⓐ $\frac{1}{4}$
ⓑ 0.25

Exercise:

Problem:

Javier will choose one digit at random from 0 through 9. The probability he will choose 3 is 10%.

Exercise:

Problem:

According to the local weather report, the probability of thunderstorms in New York City on July 15 is 60%.

Solution:

ⓐ $\frac{3}{5}$
ⓑ 0.6

Exercise:

Problem:

A club sells 50 tickets to a raffle. Osbaldo bought one ticket. The probability he will win the raffle is 2%.

Convert Decimals and Fractions to Percents

In the following exercises, convert each decimal to a percent.

Exercise:

Problem: 0.01

Solution:

1%

Exercise:

Problem: 0.03

Exercise:

Problem: 0.18

Solution:

18%

Exercise:

Problem: 0.15

Exercise:

Problem: 1.35

Solution:

135%

Exercise:

Problem: 1.56

Exercise:

Problem: 3

Solution:

300%

Exercise:

Problem: 4

Exercise:

Problem: 0.009

Solution:

0.9%

Exercise:

Problem: 0.008

Exercise:

Problem: 0.0875

Solution:

8.75%

Exercise:

Problem: 0.0625

Exercise:

Problem: 1.5

Solution:

150%

Exercise:

Problem: 2.2

Exercise:

Problem: 2.254

Solution:

225.4%

Exercise:

Problem: 2.317

In the following exercises, convert each fraction to a percent.

Exercise:

Problem: $\frac{1}{4}$

Solution:

25%

Exercise:

Problem: $\frac{1}{5}$

Exercise:

Problem: $\frac{3}{8}$

Solution:

37.5%

Exercise:

Problem: $\frac{5}{8}$

Exercise:

Problem: $\frac{7}{4}$

Solution:

175%

Exercise:

Problem: $\frac{9}{8}$

Exercise:

Problem: $6\frac{4}{5}$

Solution:

680%

Exercise:

Problem: $5\frac{1}{4}$

Exercise:

Problem: $\frac{5}{12}$

Solution:

41.7%

Exercise:

Problem: $\frac{11}{12}$

Exercise:

Problem: $2\frac{2}{3}$

Solution:

$266.\bar{6}\%$

Exercise:

Problem: $1\frac{2}{3}$

Exercise:

Problem: $\frac{3}{7}$

Solution:

42.9%

Exercise:

Problem: $\frac{6}{7}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

55.6%

Exercise:

Problem: $\frac{4}{9}$

In the following exercises, convert each fraction to a percent.

Exercise:

Problem: $\frac{1}{4}$ of washing machines needed repair.

Solution:

25%

Exercise:

Problem: $\frac{1}{5}$ of dishwashers needed repair.

In the following exercises, convert each fraction to a percent.

Exercise:

Problem:

According to the National Center for Health Statistics, in 2012, $\frac{7}{20}$ of American adults were obese.

Solution:

35%

Exercise:

Problem:

The U.S. Census Bureau estimated that in 2013, 85% of Americans lived in the same house as they did 1 year before.

In the following exercises, complete the table.

Exercise:

Problem:

Fraction	**Decimal**	**Percent**
$\frac{1}{2}$		
	0.45	
		18%
$\frac{1}{3}$		
	0.0008	
2		

Exercise:

Problem:

Fraction	**Decimal**	**Percent**
$\frac{1}{4}$		
	0.65	

Fraction	Decimal	Percent
		22%
$\frac{2}{3}$		
	0.0004	
3		

Everyday Math

Exercise:

Problem:

Sales tax Felipa says she has an easy way to estimate the sales tax when she makes a purchase. The sales tax in her city is 9.05%. She knows this is a little less than 10%.

ⓐ Convert 10% to a fraction.
ⓑ Use your answer from ⓐ to estimate the sales tax Felipa would pay on a \$95 dress.

Solution:

ⓐ $\frac{1}{10}$
ⓑ approximately $\$9.50$

Exercise:

Problem:

Savings Ryan has 25% of each paycheck automatically deposited in his savings account.

ⓐ Write 25% as a fraction.
ⓑ Use your answer from ⓐ to find the amount that goes to savings from Ryan's $\$2,400$ paycheck.

Exercise:

Problem:

Amelio is shopping for textbooks online. He found three sellers that are offering a book he needs for the same price, including shipping. To decide which seller to buy from he is comparing their customer satisfaction ratings. The ratings are given in the chart.

Seller	**Rating**
A	$4/5$
B	$3.5/4$
C	85%

Exercise:

Problem: Write seller C’s rating as a fraction and a decimal.

Solution:

$\frac{17}{20}$; 0.85

Exercise:

Problem: Write seller B’s rating as a percent and a decimal.

Exercise:

Problem: Write seller A’s rating as a percent and a decimal.

Solution:

80%; 0.8

Exercise:

Problem: Which seller should Amelio buy from and why?

Writing Exercises

Exercise:

Problem:

Convert $25\%, 50\%, 75\%,$ and 100% to fractions. Do you notice a pattern? Explain what the pattern is.

Solution:

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1.$

Exercise:

Problem:

Convert $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10},$ and $\frac{9}{10}$ to percents. Do you notice a pattern? Explain what the pattern is.

Exercise:

Problem:

When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.

Solution:

The Szetos sold their home for five times what they paid 30 years ago.

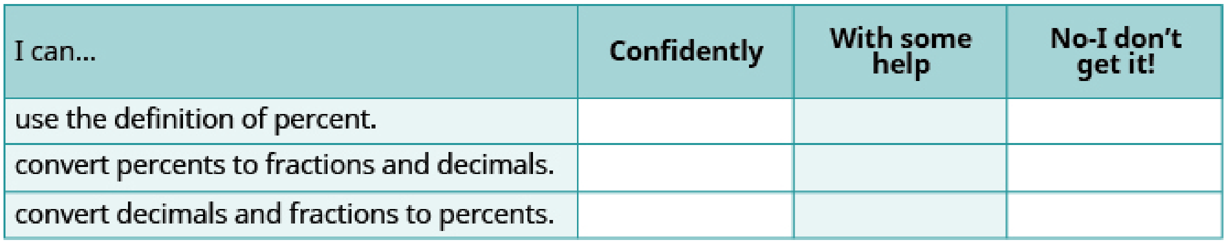
Exercise:

Problem:

According to cnn.com, cell phone use in 2008 was 600% of what it had been in 2001. Explain what 600% means in this context.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use the definition of percent.			
convert percents to fractions and decimals.			
convert decimals and fractions to percents.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don’t get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

percent	A perfecnt is a ratio whose denominator is 100.

Solve Percent Applications
By the end of this section, you will be able to:

- Translate and solve basic percent equations
- Solve percent applications
- Find percent increase and percent decrease
- Solve simple interest applications
- Solve applications with discount or mark-up

Note:
Before you get started, take this readiness quiz.

1. Convert 4.5% to a decimal.
   If you missed this problem, review [link].
2. Convert 0.6 to a percent.
   If you missed this problem, review [link].
3. Round 0.875 to the nearest hundredth.
   If you missed this problem, review [link].
4. Multiply (4.5)(2.38).
   If you missed this problem, review [link].
5. Solve $3.5 = 0.7n$.
   If you missed this problem, review [link].
6. Subtract $50 - 37.45$.
   If you missed this problem, review [link].

Translate and Solve Basic Percent Equations

We will solve percent equations using the methods we used to solve equations with fractions or decimals. Without the tools of algebra, the best method available to solve percent problems was by setting them up as proportions. Now as an algebra student, you can just translate English sentences into algebraic equations and then solve the equations.

We can use any letter you like as a variable, but it is a good idea to choose a letter that will remind us of what you are looking for. We must be sure to change the given percent to a decimal when we put it in the equation.

Example:
Exercise:

Problem: Translate and solve: What number is 35% of 90?

Solution:
Solution

	$\underbrace{\text{What number}}\ \underbrace{\text{is}}\ \underbrace{35\%}\ \underbrace{\text{of}}\ \underbrace{90?}$
Translate into algebra. Let n= the number.	$n = 0.35 \cdot 90$
Remember "of" means multiply, "is" means equals.	
Multiply.	$n = 31.5$
	31.5 is 35$\%$ of 90

Note:
Exercise:

Problem: Translate and solve:

What number is 45% of 80?

Solution:

36

Note:
Exercise:

Problem: Translate and solve:

What number is 55% of 60?

Solution:

33

We must be very careful when we translate the words in the next example. The unknown quantity will not be isolated at first, like it was in [link]. We will again use direct translation to write the equation.

Example:
Exercise:

Problem: Translate and solve: 6.5% of what number is $1.17?

Solution:
Solution

	$\underbrace{6.5\%}\ \underbrace{\text{of}}\ \underbrace{\text{what number}}\ \underbrace{\text{is}}\ \underbrace{\$1.17?}$
Translate. Let $n =$ the number.	$0.065 \cdot n = 1.17$
Multiply.	$0.065n = 1.17$
Divide both sides by 0.065 and simplify.	$n = 18$
	6.5$\%$ of \$18 is \$1.17

Note:
Exercise:

Problem: Translate and solve:

7.5% of what number is $1.95?

Solution:

$26

Note:
Exercise:

Problem: Translate and solve:

8.5% of what number is \$3.06?

Solution:

\$36

In the next example, we are looking for the percent.

Example:
Exercise:

Problem: Translate and solve: 144 is what percent of 96?

Solution:
Solution

	$\underbrace{144}_{}\quad\underbrace{\text{is}}_{}\quad\underbrace{\text{what percent}}_{}\quad\underbrace{\text{of}}_{}\quad\underbrace{96?}_{}$
Translate into algebra. Let $p =$ the percent.	$144 \quad = \quad p \quad \cdot \quad 96$
Multiply.	$144 = 96p$
Divide by 96 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	144 is 150% of 96

Note that we are asked to find percent, so we must have our final result in percent form.

Note:

Exercise:

Problem: Translate and solve:

110 is what percent of 88?

Solution:

125%

Note:
Exercise:

Problem: Translate and solve:

126 is what percent of 72?

Solution:

175%

Solve Applications of Percent

Many applications of percent—such as tips, sales tax, discounts, and interest—occur in our daily lives. To solve these applications we'll translate to a basic percent equation, just like those we solved in previous examples. Once we translate the sentence into a percent equation, we know how to solve it.

We will restate the problem solving strategy we used earlier for easy reference.

Note:
Use a Problem-Solving Strategy to Solve an Application.

Readthe problem. Make sure all the words and ideas are understood.
Identifywhat we are looking for.
Namewhat we are looking for. Choose a variable to represent that quantity.
Translateinto an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
Solvethe equation using good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications will involve everyday situations, you can rely on your own experience.

Example:
Exercise:

Problem:

Dezohn and his girlfriend enjoyed a nice dinner at a restaurant and his bill was $68.50. He wants to leave an 18% tip. If the tip will be 18% of the total bill, how much tip should he leave?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the amount of tip should Dezohn leave
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let t = amount of tip.
Step 4. Translate into an equation.		The tip is 18% of the total bill.
Write a sentence that gives the information to find it.		$\underbrace{\text{The tip}}\ \underbrace{\text{is}}\ \underbrace{18\%}\ \underbrace{\text{of}}\ \underbrace{\$68.50}$
Translate the sentence into an equation.		$t = 0.18 \cdot 68.50$
Step 5. Solve the equation. Multiply.		$t = 12.33$
Step 6. Check. Does this make sense?		
Yes, 20% of $70 is $14.		
Step 7. Answer the question with a complete sentence.		Dezohn should leave a tip of $12.33.

Notice that we used *t* to represent the unknown tip.

Note:
Exercise:

Problem:

Cierra and her sister enjoyed a dinner in a restaurant and the bill was $81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Solution:

$14.67

Note:
Exercise:

Problem:

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $14.40, how much will she leave for the tip?

Solution:

$2.16

Example:
Exercise:

Problem:

The label on Masao’s breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the total amount of potassium that is

		recommended
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let a = total amount of potassium.
Step 4. Translate. Write a sentence that gives the information to find it.		$\underbrace{\text{85 mg}}\ \underbrace{\text{is}}\ \underbrace{2\%}\ \underbrace{\text{of the}}\ \underbrace{\text{total amount}}$
Translate into an equation.		$85 = 0.02 \cdot a$
Step 5. Solve the equation.		$4{,}250 = a$
Step 6. Check. Does this make sense?		
Yes, 2% is a small percent and 85 is a small part of 4,250.		
Step 7. Answer the question with a complete sentence.		The amount of potassium that is recommended is 4,250 mg.

Note:
Exercise:

Problem:

One serving of wheat square cereal has seven grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution:

25 grams

Note:
Exercise:

Problem:

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution:

2,375 mg

Example:
Exercise:

Problem:

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the percent of the total calories from fat
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $p =$ percent of fat.
Step 4. Translate. Write a sentence that gives the information to find it.		$\underbrace{\text{What percent}}\ \underbrace{\text{of}}\ \underbrace{480}\ \underbrace{\text{is}}\ \underbrace{240?}$
Translate into an equation.		$p \cdot 480 = 240$
Step 5. Solve the equation.		$480p = 240$
Divide by 480.		$p = 0.5$
Put in a percent form.		$p = 50\%$
Step 6. Check. Does this make sense?		
Yes, 240 is half of 480, so 50% makes sense.		
Step 7. Answer the question with a complete		Of the total calories in each

sentence.		brownie, 50% is fat.

Note:
Exercise:

Problem: Solve. Round to the nearest whole percent.

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Solution:

26%

Note:
Exercise:

Problem: Solve. Round to the nearest whole percent.

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat?

Solution:

40%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the percent increase, first we find the amount of increase, the difference of the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

Note:
Find the Percent Increase.

Find the amount of increase.$\text{new amount} - \text{original amount} = \text{increase}$
Find the percent increase.The increase is what percent of the original amount?

Example:
Exercise:

Problem:

In 2011, the California governor proposed raising community college fees from $26 a unit to $36 a unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the percent increase
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $p =$ the percent.
Step 4. Translate. Write a sentence that gives the information to find it.		
First find the amount of increase.		new amount − original amount = increase
		$36 - 26 = 10$
Find the percent.		Increase is what percent of the original amount?
		$\underbrace{10}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{26?}$
Translate into an equation.		$10 = p \cdot 26$
Step 5. Solve the equation.		$10 = 26p$
Divide by 26.		$0.384 = p$
Change to percent form; round to the nearest tenth.		$38.4\% = p$

Step 6. Check. Does this make sense?		
Yes, 38.4% is close to $\frac{1}{3}$, and 10 is close to $\frac{1}{3}$ of 26.		
Step 7. Answer the question with a complete sentence.		The new fees represent a 38.4% increase over the old fees.

Notice that we rounded the division to the nearest thousandth in order to round the percent to the nearest tenth.

Note:
Exercise:

Problem: Find the percent increase. (Round to the nearest tenth of a percent.)

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Solution:

8.8%

Note:
Exercise:

Problem: Find the percent increase.

In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was $2.25.

Solution:

50%

Finding the percent decrease is very similar to finding the percent increase, but now the amount of decrease is the difference of the original amount and the new amount. Then we find what percent the amount of decrease is of the original amount.

Note:
Find the Percent Decrease.

Find the amount of decrease. $\text{original amount} - \text{new amount} = \text{decrease}$

Find the percent decrease.The decrease is what percent of the original amount?

Example:
Exercise:

Problem:

The average price of a gallon of gas in one city in June 2014 was \$3.71. The average price in that city in July was \$3.64. Find the percent decrease.

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the percent decrease
Step 3. Name what we are looking for.		
Choose a variable to represent that quantity.		Let $p =$ the percent decrease.
Step 4. Translate. Write a sentence that gives the information to find it.		
First find the amount of decrease.		$3.71 - 3.64 = 0.07$
Find the percent.		Decrease is what percent of the original amaount?
		$\underbrace{0.07}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{3.71}?$
Translate into an equation.		$0.07 = p \cdot 3.71$
Step 5. Solve the equation.		$0.07 = 3.71p$
Divide by 3.71.		$0.019 = p$
Change to percent form; round to the nearest		

tenth.		$1.9\% = p$
Step 6. Check. Does this make sense?		
Yes, if the original price was $4, a 2% decrease would be 8 cents.		
Step 7. Answer the question with a complete sentence.		The price of gas decreased 1.9%.

Note:
Exercise:

Problem: Find the percent decrease. (Round to the nearest tenth of a percent.)

The population of North Dakota was about 672,000 in 2010. The population is projected to be about 630,000 in 2020.

Solution:

6.3%

Note:
Exercise:

Problem: Find the percent decrease.

Last year, Sheila’s salary was $42,000. Because of furlough days, this year, her salary was $37,800.

Solution:

10%

Solve Simple Interest Applications

Do you know that banks pay you to keep your money? The money a customer puts in the bank is called the **principal**, P, and the money the bank pays the customer is called the **interest**. The interest is computed as a certain percent of the principal; called the **rate of interest**, r. We usually express rate of interest as a percent per year, and we calculate it by using the decimal equivalent of the percent. The variable t, (for *time*) represents the number of years the money is in the account.

To find the interest we use the simple interest formula, $I = Prt$.

Note:

Simple Interest

If an amount of money, P, called the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I, earned is

Equation:

$$I = Prt \qquad \text{where} \qquad \begin{array}{rcl} I & = & \text{interest} \\ P & = & \text{principal} \\ r & = & \text{rate} \\ t & = & \text{time} \end{array}$$

Interest earned according to this formula is called **simple interest**.

Interest may also be calculated another way, called compound interest. This type of interest will be covered in later math classes.

The formula we use to calculate simple interest is $I = Prt$. To use the formula, we substitute in the values the problem gives us for the variables, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

Example:

Exercise:

Problem:

Nathaly deposited \$12,500 in her bank account where it will earn 4% interest. How much interest will Nathaly earn in 5 years?

Equation:

$$\begin{array}{rcl} I & = & ? \\ P & = & \$12{,}500 \\ r & = & 4\% \\ t & = & 5\text{ years} \end{array}$$

Solution:

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of interest earned
Step 3. Name what we are looking for. Choose a variable to represent that quantity	Let $I =$ the amount of interest.
Step 4. Translate into an equation.	
Write the formula.	$I = Prt$
Substitute in the given information.	$I = (12{,}500)(.04)(5)$
Step 5. Solve the equation.	$I = 2{,}500$
Step 6. Check: Does this make sense?	
Is $2,500 is a reasonable interest on $12,500? Yes.	
Step 7. Answer the question with a complete sentence.	The interest is $2,500.

Note:
Exercise:

Problem:

Areli invested a principal of $950 in her bank account with interest rate 3%. How much interest did she earn in 5 years?

Solution:

$142.50

Note:
Exercise:

Problem:

Susana invested a principal of $36,000 in her bank account with interest rate 6.5%. How much interest did she earn in 3 years?

Solution:

$7,020

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we don’t know the rate. To find the rate, we use the simple interest formula, substitute in the given values for the principal and time, and then solve for the rate.

Example:
Exercise:

Problem:

Loren loaned his brother \$3,000 to help him buy a car. In 4 years his brother paid him back the \$3,000 plus \$660 in interest. What was the rate of interest?

Equation:

$$\begin{aligned} I &= \$660 \\ P &= \$3{,}000 \\ r &= ? \\ t &= 4 \text{ years} \end{aligned}$$

Solution:
Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for. the rate of interest

Step 3. Name what we are looking for. Choose a variable to represent that quantity. Let r = rate of interest.

Step 4. Translate into an equation.

Write the formula. $I = Prt$

Substitute in the given information. $660 = (3{,}000)r(4)$

Step 5. Solve the equation. $660 = (12{,}000)r$

Divide. $0.055 = r$

Change to percent form. $5.5\% = r$

Step 6. Check: Does this make sense?

$$\begin{aligned} I &= Prt \\ 660 &\overset{?}{=} (3{,}000)(0.055)(4) \\ 660 &= 660\checkmark \end{aligned}$$

Step 7. Answer the question with a complete sentence. The rate of interest was 5.5%.

Notice that in this example, Loren’s brother paid Loren interest, just like a bank would have paid interest if Loren invested his money there.

Note:
Exercise:

Problem:

Jim loaned his sister \$5,000 to help her buy a house. In 3 years, she paid him the \$5,000, plus \$900 interest. What was the rate of interest?

Solution:

6%

Note:
Exercise:

Problem:

Hang borrowed \$7,500 from her parents to pay her tuition. In 5 years, she paid them \$1,500 interest in addition to the \$7,500 she borrowed. What was the rate of interest?

Solution:

4%

Example:
Exercise:

Problem:

Eduardo noticed that his new car loan papers stated that with a 7.5% interest rate, he would pay \$6,596.25 in interest over 5 years. How much did he borrow to pay for his car?

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount borrowed (the principal)
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let P = principal borrowed.
Step 4. Translate into an equation.	
Write the formula.	$I = Prt$
Substitute in the given information.	$6{,}596.25 = P(0.075)(5)$
Step 5. Solve the equation.	$6{,}596.25 = 0.375P$
Divide.	$17{,}590 = P$
Step 6. Check: Does this make sense? $I = Prt$ $6{,}596.25 \overset{?}{=} (17{,}590)(0.075)(5)$ $6{,}596.25 = 6{,}596.25 \checkmark$	
Step 7. Answer the question with a complete sentence.	The principal was $17,590.

Note:
Exercise:

Problem:

Sean’s new car loan statement said he would pay $4,866.25 in interest from an interest rate of 8.5% over 5 years. How much did he borrow to buy his new car?

Solution:

$11,450

Note:
Exercise:

Problem:

In 5 years, Gloria’s bank account earned $2,400 interest at 5%. How much had she deposited in the account?

Solution:

$9,600

Solve Applications with Discount or Mark-up

Applications of discount are very common in retail settings. When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the amount of the discount. To determine the **amount of discount**, we multiply the discount rate by the original price.

We summarize the discount model in the box below.

Note:
Discount
Equation:

$$\begin{aligned}&\text{amount of discount} = \text{discount rate} \times \text{original price}\\&\text{sale price} = \text{original price} - \text{amount of discount}\end{aligned}$$

Keep in mind that the sale price should always be less than the original price.

Example:
Exercise:

Problem:

Elise bought a dress that was discounted 35% off of the original price of $140. What was ⓐ the amount of discount and ⓑ the sale price of the dress?

Solution:
Solution

ⓐ

$$\begin{aligned}\text{Original price} &= \$140\\ \text{Discount rate} &= 35\%\\ \text{Discount} &= ?\end{aligned}$$

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of discount
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $d=$ the amount of discount.
Step 4. Translate into an equation. Write a sentence that gives the information to find it. Translate into an equation.	The discount is 35% of \$140. $d=0.35(140)$
Step 5. Solve the equation.	$d=49$
Step 6. Check: Does this make sense? Is a \$49 discount reasonable for a \$140 dress? Yes.	
Step 7. Write a complete sentence to answer the question.	The amount of discount was \$49.

ⓑ
Read the problem again.

Step 1. Identify what we are looking for.		the sale price of the dress
Step 2. Name what we are looking for.		
Choose a variable to represent that quantity.		Let $s=$ the sale price.
Step 3. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The sale price}}\ \underbrace{\text{is}}\ \underbrace{\text{the \$140}}\ \underbrace{\text{minus}}\ \underbrace{\text{the \$49 discount}}$
Translate into an equation.		$s \quad = \quad 140 \quad - \quad 49$
Step 4. Solve the equation.		$s=91$

Step 5. Check. Does this make sense?		
Is the sale price less than the original price?		
Yes, $91 is less than $140.		
Step 6. Answer the question with a complete sentence.		The sale price of the dress was $91.

Note:
Exercise:

Problem: Find ⓐ the amount of discount and ⓑ the sale price:

Sergio bought a belt that was discounted 40% from an original price of $29.

Solution:

ⓐ $11.60 ⓑ $17.40

Note:
Exercise:

Problem: Find ⓐ the amount of discount and ⓑ the sale price:

Oscar bought a barbecue that was discounted 65% from an original price of $395.

Solution:

ⓐ $256.75 ⓑ $138.25

There may be times when we know the original price and the sale price, and we want to know the discount rate. To find the discount rate, first we will find the amount of discount and then use it to compute the rate as a percent of the original price. [link] will show this case.

Example:
Exercise:

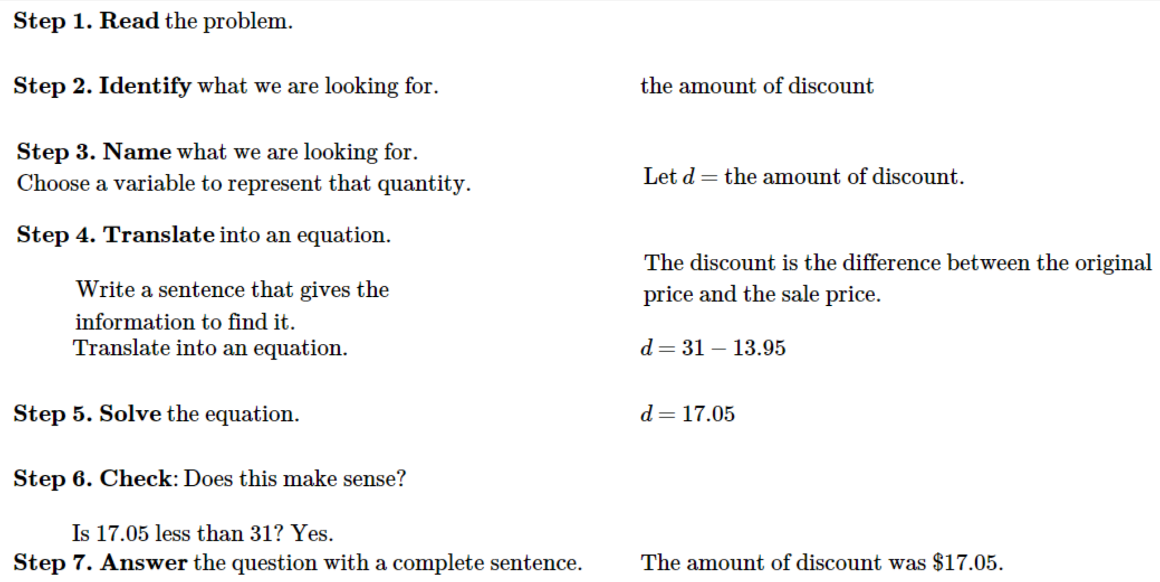
Problem:

Jeannette bought a swimsuit at a sale price of \$13.95. The original price of the swimsuit was \$31. Find the ⓐ amount of discount and ⓑ discount rate.

Solution:
Solution

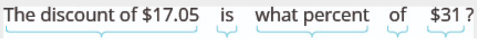
ⓐ

$$\begin{aligned} \text{Original price} &= \$31 \\ \text{Discount} &= ? \\ \text{Sale Price} &= \$13.95 \end{aligned}$$



Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of discount
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $d =$ the amount of discount.
Step 4. Translate into an equation.	
Write a sentence that gives the information to find it.	The discount is the difference between the original price and the sale price.
Translate into an equation.	$d = 31 - 13.95$
Step 5. Solve the equation.	$d = 17.05$
Step 6. Check: Does this make sense?	
Is 17.05 less than 31? Yes.	
Step 7. Answer the question with a complete sentence.	The amount of discount was \$17.05.

ⓑ
Read the problem again.

Step 1. Identify what we are looking for.		the discount rate
Step 2. Name what we are looking for.		
Choose a variable to represent it.		Let $r =$ the discount rate.
Step 3. Translate into an equation.		
Write a sentence that gives the		 $\underbrace{\text{The discount of \$17.05}}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{\text{\$31}}\ ?$

information to find it.		
Translate into an equation.		$17.05 = r \cdot 31$
Step 4. Solve the equation.		$17.05 = 31r$
Divide both sides by 31.		$0.55 = r$
Change to percent form.		$r = 55\%$
Step 5. Check. Does this make sense?		
Is $17.05 equal to 55% of $31?		
$17.05 \overset{?}{=} 0.55(31)$		
$17.05 = 17.05\checkmark$		
Step 6. Answer the question with a complete sentence.		The rate of discount was 55%.

Note:
Exercise:

Problem: Find ⓐ the amount of discount and ⓑ the discount rate.

Lena bought a kitchen table at the sale price of $375.20. The original price of the table was $560.

Solution:

ⓐ $184.80 ⓑ 33%

Note:
Exercise:

Problem: Find ⓐ the amount of discount and ⓑ the discount rate.

Nick bought a multi-room air conditioner at a sale price of $340. The original price of the air conditioner was $400.

Solution:

ⓐ \$60 ⓑ 15%

Applications of mark-up are very common in retail settings. The price a retailer pays for an item is called the **original cost**. The retailer then adds a **mark-up** to the original cost to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

We summarize the mark-up model in the box below.

Note:
Mark-Up
Equation:

$$\begin{aligned}&\text{amount of mark-up} = \text{mark-up rate } \times \text{ original cost}\\&\text{list price} = \text{original cost} + \text{amount of mark up}\end{aligned}$$

Keep in mind that the list price should always be more than the original cost.

Example:
Exercise:

Problem:

Adam’s art gallery bought a photograph at original cost \$250. Adam marked the price up 40%. Find the ⓐ amount of mark-up and ⓑ the list price of the photograph.

Solution:
Solution

ⓐ

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the amount of mark-up
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $m =$ the amount of markup.

Step 4. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The mark-up}}\ \underbrace{\text{is}}\ \underbrace{\text{40\%}}\ \underbrace{\text{of}}\ \underbrace{\text{the \$250 original cost}}$
Translate into an equation.		$m = 0.40 \cdot 250$
Step 5. Solve the equation.		$m = 100$
Step 6. Check. Does this make sense?		
Yes, 40% is less than one-half and 100 is less than half of 250.		
Step 7. Answer the question with a complete sentence.		The mark-up on the phtograph was $100.

ⓑ

Step 1. Read the problem again.		
Step 2. Identify what we are looking for.		the list price
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $p =$ the list price.
Step 4. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The list price}}\ \underbrace{\text{is}}\ \underbrace{\text{original cost}}\ \underbrace{\text{plus}}\ \underbrace{\text{the mark-up}}$
Translate into an equation.		$p = 250 + 100$
Step 5. Solve the equation.		$p = 350$
Step 6. Check. Does this make sense?		
Is the list price more than the net price?		

Is \$350 more than \$250? Yes.		
Step 7. Answer the question with a complete sentence.		The list price of the photograph was \$350.

Note:
Exercise:

Problem: Find ⓐ the amount of mark-up and ⓑ the list price.

Jim’s music store bought a guitar at original cost \$1,200. Jim marked the price up 50%.

Solution:

ⓐ \$600 ⓑ \$1,800

Note:
Exercise:

Problem: Find ⓐ the amount of mark-up and ⓑ the list price.

The Auto Resale Store bought Pablo’s Toyota for \$8,500. They marked the price up 35%.

Solution:

ⓐ \$2,975 ⓑ \$11,475

Key Concepts

- **Percent Increase** To find the percent increase:

  Find the amount of increase. $\text{increase} = \text{new amount} - \text{original amount}$
  Find the percent increase. Increase is what percent of the original amount?

- **Percent Decrease** To find the percent decrease:

  Find the amount of decrease. $\text{decrease} = \text{original amount} - \text{new amount}$
  Find the percent decrease. Decrease is what percent of the original amount?

- **Simple Interest** If an amount of money, P, called the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I earned is
  Equation:

$$\begin{array}{rrcl} & I & = & Prt \\ \text{where} & I & = & \text{interest} \\ & P & = & \text{principal} \\ & r & = & \text{rate} \\ & t & = & \text{time} \end{array}$$

- **Discount**
  - amount of discount is discount rate · original price
  - sale price is original price – discount
- **Mark-up**
  - amount of mark-up is mark-up rate · original cost
  - list price is original cost + mark up

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

Exercise:

Problem: What number is 45% of 120?

Solution:

54

Exercise:

Problem: What number is 65% of 100?

Exercise:

Problem: What number is 24% of 112?

Solution:

26.88

Exercise:

Problem: What number is 36% of 124?

Exercise:

Problem: 250% of 65 is what number?

Solution:

162.5

Exercise:

Problem: 150% of 90 is what number?

Exercise:

Problem: 800% of 2250 is what number?

Solution:

18,000

Exercise:

Problem: 600% of 1740 is what number?

Exercise:

Problem: 28 is 25% of what number?

Solution:

112

Exercise:

Problem: 36 is 25% of what number?

Exercise:

Problem: 81 is 75% of what number?

Solution:

108

Exercise:

Problem: 93 is 75% of what number?

Exercise:

Problem: 8.2% of what number is $2.87?

Solution:

$35

Exercise:

Problem: 6.4% of what number is $2.88?

Exercise:

Problem: 11.5% of what number is $108.10?

Solution:

$940

Exercise:

Problem: 12.3% of what number is $92.25?

Exercise:

Problem: What percent of 260 is 78?

Solution:

30%

Exercise:

Problem: What percent of 215 is 86?

Exercise:

Problem: What percent of 1500 is 540?

Solution:

36%

Exercise:

Problem: What percent of 1800 is 846?

Exercise:

Problem: 30 is what percent of 20?

Solution:

150%

Exercise:

Problem: 50 is what percent of 40?

Exercise:

Problem: 840 is what percent of 480?

Solution:

175%

Exercise:

Problem: 790 is what percent of 395?

Solve Percent Applications

In the following exercises, solve.

Exercise:

Problem:

Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

Solution:

$11.88

Exercise:

Problem:

When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?

Exercise:

Problem:

Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $2165. How much money was deposited to Trong’s savings account?

Solution:

$259.80

Exercise:

Problem:

Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $1,485. How much did Cherise deposit into her retirement account?

Exercise:

Problem:

One serving of oatmeal has eight grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution:

24.2 g

Exercise:

Problem:

One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

Exercise:

Problem:

A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution:

2407 mg

Exercise:

Problem:

A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

Exercise:

Problem:

After 3 months on a diet, Lisa had lost 12% of her original weight. She lost 21 pounds. What was Lisa's original weight?

Solution:

175 lb.

Exercise:

Problem:

Tricia got a 6% raise on her weekly salary. The raise was $30 per week. What was her original salary?

Exercise:

Problem:

Yuki bought a dress on sale for $72. The sale price was 60% of the original price. What was the original price of the dress?

Solution:

$120

Exercise:

Problem:

Kim bought a pair of shoes on sale for $40.50. The sale price was 45% of the original price. What was the original price of the shoes?

Exercise:

Problem: Tim left a $9 tip for a $50 restaurant bill. What percent tip did he leave?

Solution:

18%

Exercise:

Problem: Rashid left a $15 tip for a $75 restaurant bill. What percent tip did he leave?

Exercise:

Problem:

The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Solution:

45%

Exercise:

Problem:

The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

Exercise:

Problem:

Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Solution:

25%

Exercise:

Problem:

Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, solve.

Exercise:

Problem: Tamanika got a raise in her hourly pay, from $15.50 to $17.36. Find the percent increase.

Solution:

12%

Exercise:

Problem: Ayodele got a raise in her hourly pay, from $24.50 to $25.48. Find the percent increase.

Exercise:

Problem:

Annual student fees at the University of California rose from about $4,000 in 2000 to about $12,000 in 2010. Find the percent increase.

Solution:

200%

Exercise:

Problem: The price of a share of one stock rose from $12.50 to $50. Find the percent increase.

Exercise:

Problem:

According to *Time* magazine annual global seafood consumption rose from 22 pounds per person in the 1960s to 38 pounds per person in 2011. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:

72.7%

Exercise:

Problem:

In one month, the median home price in the Northeast rose from $225,400 to $241,500. Find the percent increase. (Round to the nearest tenth of a percent.)

Exercise:

Problem:

A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.

Solution:

2.5%

Exercise:

Problem: The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.

Exercise:

Problem:

Hernando’s salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.

Solution:

11%

Exercise:

Problem:

In 10 years, the population of Detroit fell from 950,000 to about 712,500. Find the percent decrease.

Exercise:

Problem:

In 1 month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution:

5.5%

Exercise:

Problem:

Sales of video games and consoles fell from $1,150 million to $1,030 million in 1 year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solve Simple Interest Applications

In the following exercises, solve.

Exercise:

Problem:

Casey deposited $1,450 in a bank account with interest rate 4%. How much interest was earned in two years?

Solution:

$116

Exercise:

Problem:

Terrence deposited $5,720 in a bank account with interest rate 6%. How much interest was earned in 4 years?

Exercise:

Problem:

Robin deposited $31,000 in a bank account with interest rate 5.2%. How much interest was earned in 3 years?

Solution:

$4,836

Exercise:

Problem:

Carleen deposited $16,400 in a bank account with interest rate 3.9%. How much interest was earned in 8 years?

Exercise:

Problem:

Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of interest?

Solution:

3%

Exercise:

Problem:

Kenneth loaned his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of interest?

Exercise:

Problem:

Lebron loaned his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of interest?

Solution:

3.75%

Exercise:

Problem:

Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of interest?

Exercise:

Problem:

In 10 years, a bank account that paid 5.25% earned $18,375 interest. What was the principal of the account?

Solution:

$35,000

Exercise:

Problem:

In 25 years, a bond that paid 4.75% earned $2,375 interest. What was the principal of the bond?

Exercise:

Problem:

Joshua's computer loan statement said he would pay $1,244.34 in interest for a 3-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Solution:

$3,345

Exercise:

Problem:

Margaret's car loan statement said she would pay $7,683.20 in interest for a 5-year loan at 9.8%. How much did Margaret borrow to buy the car?

Solve Applications with Discount or Mark-up

In the following exercises, find the sale price.

Exercise:

Problem:

Perla bought a cell phone that was on sale for $50 off. The original price of the cell phone was $189.

Solution:

$139

Exercise:

Problem: Sophie saw a dress she liked on sale for $15 off. The original price of the dress was $96.

Exercise:

Problem:

Rick wants to buy a tool set with original price \$165. Next week the tool set will be on sale for \$40 off.

Solution:

\$125

Exercise:

Problem:

Angelo's store is having a sale on televisions. One television, with original price \$859, is selling for \$125 off.

In the following exercises, find ⓐ the amount of discount and ⓑ the sale price.

Exercise:

Problem: Janelle bought a beach chair on sale at 60% off. The original price was \$44.95.

Solution:

ⓐ \$26.97 ⓑ \$17.98

Exercise:

Problem: Errol bought a skateboard helmet on sale at 40% off. The original price was \$49.95.

Exercise:

Problem:

Kathy wants to buy a camera that lists for \$389. The camera is on sale with a 33% discount.

Solution:

ⓐ \$128.37 ⓑ \$260.63

Exercise:

Problem: Colleen bought a suit that was discounted 25% from an original price of \$245.

Exercise:

Problem:

Erys bought a treadmill on sale at 35% off. The original price was \$949.95 (round to the nearest cent.)

Solution:

ⓐ \$332.48 ⓑ \$617.47

Exercise:

Problem:

Jay bought a guitar on sale at 45% off. The original price was $514.75 (round to the nearest cent.)

In the following exercises, find ⓐ the amount of discount and ⓑ the discount rate. (Round to the nearest tenth of a percent if needed.)

Exercise:

Problem:

Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

Solution:

ⓐ $576 ⓑ 30%

Exercise:

Problem:

Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

Exercise:

Problem:

Patty bought a baby stroller on sale for $301.75. The original price of the stroller was $355.

Solution:

ⓐ $53.25 ⓑ 15%

Exercise:

Problem: Bill found a book he wanted on sale for $20.80. The original price of the book was $32.

Exercise:

Problem:

Nikki bought a patio set on sale for $480. The original price was $850. To the nearest tenth of a percent, what was the rate of discount?

Solution:

ⓐ $370 ⓑ 43.5%

Exercise:

Problem:

Stella bought a dinette set on sale for $725. The original price was $1,299. To the nearest tenth of a percent, what was the rate of discount?

In the following exercises, find ⓐ the amount of the mark-up and ⓑ the list price.

Exercise:

Problem:

Daria bought a bracelet at original cost $16 to sell in her handicraft store. She marked the price up 45%.

Solution:

ⓐ $7.20 ⓑ $23.20

Exercise:

Problem:

Regina bought a handmade quilt at original cost $120 to sell in her quilt store. She marked the price up 55%.

Exercise:

Problem:

Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up.

Solution:

ⓐ $0.20 ⓑ $0.80

Exercise:

Problem:

Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up.

Exercise:

Problem:

Alan bought a used bicycle for $115. After re-conditioning it, he added 225% mark-up and then advertised it for sale.

Solution:

ⓐ $258.75 ⓑ $373.75

Exercise:

Problem:

Michael bought a classic car for $8,500. He restored it, then added 150% mark-up before advertising it for sale.

Everyday Math

Exercise:

Problem:

Leaving a Tip At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Solution:

21.2%

Exercise:

Problem:

Splitting a Bill Four friends went out to lunch and the bill came to $53.75. They decided to add enough tip to make a total of $64, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Writing Exercises

Exercise:

Problem:

Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

Solution:

The number should be greater than 44. Since 80% equals 0.8 in decimal form, 0.8 is less than one, and we must multiply the number by 0.8 to get 44, the number must be greater than 44.

Exercise:

Problem:

Without solving the problem “What is 20% of 300?” think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.

Exercise:

Problem:

After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Solution:

He meant that he should have packed half the shorts and twice the shirts.

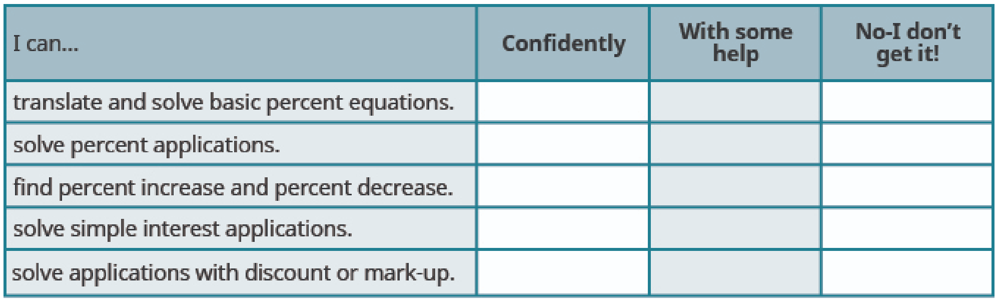
Exercise:

Problem:

Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
translate and solve basic percent equations.			
solve percent applications.			
find percent increase and percent decrease.			
solve simple interest applications.			
solve applications with discount or mark-up.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

Glossary

amount of discount

The amount of discount is the amount resulting when a discount rate is multiplied by the original price of an item.

discount rate

The discount rate is the percent used to determine the amount of a discount, common in retail settings.

interest

Interest is the money that a bank pays its customers for keeping their money in the bank.

list price

The list price is the price a retailer sells an item for.

mark-up

A mark-up is a percentage of the original cost used to increase the price of an item.

original cost

The original cost in a retail setting, is the price that a retailer pays for an item.

principal

The principal is the original amount of money invested or borrowed for a period of time at a specific interest rate.

rate of interest

The rate of interest is a percent of the principal, usually expressed as a percent per year.

simple interest

Simple interest is the interest earned according to the formula $I = Prt$.

Scientific Notation

Learning Objectives

By the end of this lesson, you will be able to

- Read and understand scientific notation.
- Convert between decimal notation and scientific notation.
- Use scientific notation on a calculator.

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on.

Consider the numbers 4000 and 0.004. We know that 4000 means $4\ \times\ 1000$ and 0.004 means $4\ \times\ \frac{1}{1000}$. If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

Equation:

$$\begin{array}{ll} 4000 & 0.004 \\ 4\ \times\ 1000 & 4\ \times\ \frac{1}{1000} \\ 4\ \times\ 10^{3} & 4\ \times\ \frac{1}{10^{3}} \\ & 4\ \times\ 10^{-3} \end{array}$$

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in *scientific notation.*

Note:

Scientific Notation

A number is expressed in **scientific notation** when it is of the form

Equation:

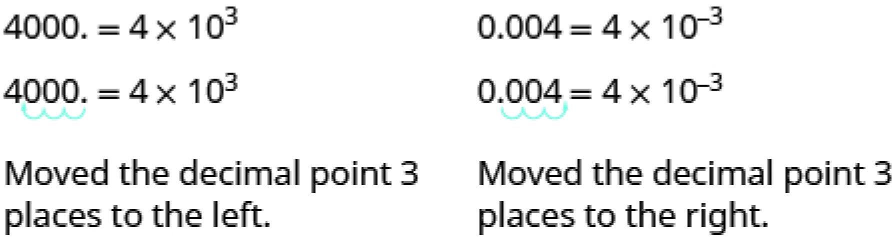
$$a \times 10^n$$

where $a \geq 1$ and $a < 10$ and n is an integer.

It is customary in scientific notation to use $\times$ as the multiplication sign, even though we avoid using this sign elsewhere in algebra.

Scientific notation is a useful way of writing very large or very small numbers. It is used often in the sciences to make calculations easier.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.



$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
Moved the decimal point 3 places to the left.	Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor, 4, by itself.

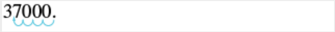
- The power of 10 is positive when the number is larger than $1: 4000 = 4 \times 10^3$.
- The power of 10 is negative when the number is between 0 and $1: 0.004 = 4 \times 10^{-3}$.

Example:

Exercise:

Problem: Write $37{,}000$ in scientific notation.

Solution:

Step 1: Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	
Step 2: Count the number of decimal places, n, that the decimal point was moved.	3.70000 4 places
Step 3: Write the number as a product with a power of 10.	3.7×10^4
If the original number is: • greater than 1, the power of 10 will be 10^n. • between 0 and 1, the power of 10 will be $10^{-\mathrm{n}}$	
Step 4: Check.	
10^4 is 10,000 and 10,000 times 3.7 will be 37,000.	
	$37{,}000 = 3.7 \times 10^4$

Example:
Exercise:

Problem: Write in scientific notation: 96,000.

Solution:

9.6×10^4

Note:
Try It
Exercise:

Problem: Write in scientific notation: 48,300.

Solution:

4.83×10^4

Note:
Convert from decimal notation to scientific notation

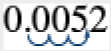
- Move the decimal point so that the first factor is greater than or equal to 1 but less than $10.$
- Count the number of decimal places, $n,$ that the decimal point was moved.
- Write the number as a product with a power of $10.$
  - If the original number is:
  - greater than 1, the power of 10 will be 10^n.
  - between 0 and 1, the power of 10 will be 10^{-n}.

Check.

Example:
Exercise:

Problem: Write in scientific notation: 0.0052.

Solution:

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}

$$5.2 \times 10^{-3}$$

$$5.2 \times \frac{1}{10^3}$$

$$5.2 \times \frac{1}{1000}$$

$$5.2 \times 0.001$$

$$0.0052$$

$$0.0052 = 5.2 \times 10^{-3}$$

Example:
Exercise:

Problem: Write in scientific notation: 0.0078.

Solution:

7.8×10^{-3}

Note:
Try It
Exercise:

Problem: Write in scientific notation: 0.0129.

Solution:

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let’s look at two numbers written in scientific notation and see.
Equation:

$$\begin{array}{ll} 9.12 \times 10^{4} & 9.12 \times 10^{-4} \\ 9.12 \times 10{,}000 & 9.12 \times 0.0001 \\ 91{,}200 & 0.000912 \end{array}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$9.12 \times 10^{4} = 91{,}200 \qquad 9.12 \times 10^{-4} = 0.000912$$

$$9.12\_\_ \times 10^{4} = 91{,}200 \qquad \_\_\_9.12 \times 10^{-4} = 0.000912$$

In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example:
Exercise:

Problem: Convert to decimal form: 6.2×10^{3}.

Solution:

Step 1: Determine the exponent, n, on the factor 10.	6×10^{3}
Step 2: Move the decimal point n places, adding zeros if needed.	6.200,
• If the exponent is positive, move the decimal point n places to the right. • If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	6,200

Step 3: Check to see if your answer makes sense.	
10^3 is 1000 and 1000 times 6.2 will be 6,200. $6.2 \times 10^3 = 6{,}200$	

Example:
Exercise:

Problem: Convert to decimal form: $1.3 \ \times \ 10^3$.

Solution:

1,300

Note:
Try It
Exercise:

Problem: Convert to decimal form: $9.25 \ \times \ 10^4$.

Solution:

92,500

Note:
Convert scientific notation to decimal form

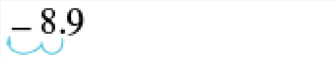
Determine the exponent, n, on the factor 10.

Move the decimal n places, adding zeros if needed.	○ If the exponent is positive, move the decimal point n places to the right. ○ If the exponent is negative, move the decimal point $\|n\|$ places to the left.
Check.	

Example:
Exercise:

Problem: Convert to decimal form: 8.9×10^{-2}.

Solution:

	8.9×10^{-2}
Determine the exponent n, on the factor 10.	The exponent is -2.
Move the decimal point 2 places to the left.	
Add zeros as needed for placeholders.	0.089
	$8.9 \times 10^{-2} = 0.089$
The Check is left to you.	

Example:
Exercise:

Problem: Convert to decimal form: 1.2×10^{-4}.

Solution:

0.00012

Note:
Try It
Exercise:

Problem: Convert to decimal form: 7.5×10^{-2}.

Solution:

0.075

Scientific Notation on a Scientific Calculator

It's important that you know how to enter numbers into a calculator that are given in scientific notation, without having to convert it to decimal notation first, as well as how to read an answer that the calculator gives in scientific notation.

On a scientific calculator, there is a button specifically meant for scientific notation. This button is either an Exp or an EE button. When you press the button an E will show on the screen (or × 10^) and the E represents × 10^, where the ^ symbol means "raised to the". You would then follow that with the exponent.

Try with your calculator: Enter in 7.15×10^3 as 7.15 Exp (or EE) 3, hit enter or the = sign, and your calculator will show 7150, which is the decimal notation.

Now enter in 7.15×10^{15} as 7.15 Exp (or EE) 15, hit enter or the = sign, and your calculator should show 7.15×10^{15} or 7.15E15. This is because this number has too many digits to show on the screen in decimal notation.

Example:
Exercise:

Problem: Multiply $(1.5 \times 10^{-7})(8.2 \times 10^{3})$ using your calculator.

Solution:

This would be entered into the calculator as (1.5 Exp (or EE) −7)(8.2 Exp (or EE) 3). The calculator should give 0.00123 as the answer.

Note:
Try It
Exercise:

Problem: Multiply $(9.9 \times 10^{5})(9.9 \times 10^{20})$ using your calculator.

Solution:

Your calculator may show you 9.801 E 26, which means 9.801×10^{26}.

Example:
Exercise:

Problem: Use the calculator to divide: $\frac{2.04\times10^{5}}{3.4\times10^{-6}}$

Solution:

This is entered as 2.04 Exp 5 ÷ 3.4 Exp -6. The answer is 6×10^{10}.

Summary

- **Convert from Decimal Notation to Scientific Notation:** To convert a decimal to scientific notation:

  Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
  Count the number of decimal places, n, that the decimal point was moved.
  Write the number as a product with a power of 10.
    - If the original number is greater than 1, the power of 10 will be 10^{n}.
    - If the original number is between 0 and 1, the power of 10 will be 10^{n}.

  Check.

- **Convert Scientific Notation to Decimal Form:** To convert scientific notation to decimal form:

  Determine the exponent, n, on the factor 10.
  Move the decimal n places, adding zeros if needed.
    - If the exponent is positive, move the decimal point n places to the right.
    - If the exponent is negative, move the decimal point $|n|$ places to the left.

Check.

Homework

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

Exercise:

Problem: 280,000

Solution:

2.8×10^5

Exercise:

Problem: 45,000

Exercise:

Problem: 1,290,000

Solution:

1.29×10^6

Exercise:

Problem: 8,750,000

Exercise:

Problem: 0.041

Solution:

4.1×10^{-2}

Exercise:

Problem: 0.036

Exercise:

Problem: 0.0000103

Solution:

1.03×10^{-5}

Exercise:

Problem: 0.00000924

Exercise:

Problem:

The population of the world on July 4, 2010 was more than 6,850,000,000.

Solution:

6.85×10^{9}

Exercise:

Problem:

The population of the United States on July 4, 2010 was almost 310,000,000.

Exercise:

Problem:

The probability of winning the 2010 Megamillions lottery is about 0.0000000057.

Solution:

5.7×10^{-9}

Exercise:

Problem: The average width of a human hair is 0.0018 centimeters.

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

Exercise:

Problem: 8.3×10^2

Solution:

830

Exercise:

Problem: 4.1×10^2

Exercise:

Problem: 1.6×10^{10}

Solution:

16,000,000,000

Exercise:

Problem: 5.5×10^8

Exercise:

Problem: 2.8×10^{-2}

Solution:

0.028

Exercise:

Problem: 3.5×10^{-2}

Exercise:

Problem: 6.15×10^{-8}

Solution:

0.0000000615

Exercise:

Problem: 1.93×10^{-5}

Exercise:

Problem:

At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$.

Solution:

$15,000,000,000,000

Exercise:

Problem:

In 2010, the number of Facebook users each day who changed their status to ‘engaged’ was 2×10^{4}.

Exercise:

Problem: The width of a proton is 1×10^{-5} of the width of an atom.

Solution:

0.00001

Exercise:

Problem:

The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}.

Multiply and Divide Using Scientific Notation and a Scientific Calculator

In the following exercises, multiply or divide, using a scientific calculator, and write your answer in decimal form.

Exercise:

Problem: $(3 \times 10^{2})(1 \times 10^{-5})$

Solution:

0.003

Exercise:

Problem: $(2 \times 10^{5})(2 \times 10^{-9})$

Exercise:

Problem: $(2.1 \times 10^{-4})(3.5 \times 10^{-2})$

Solution:

0.00000735

Exercise:

Problem: $(1.6 \times 10^{-2})(5.2 \times 10^{-6})$

Exercise:

Problem: $\frac{8 \times 10^{6}}{4 \times 10^{-1}}$

Solution:

20,000,000

Exercise:

Problem: $\frac{6 \times 10^{4}}{3 \times 10^{-2}}$

Exercise:

Problem: $\frac{5 \times 10^{-3}}{1 \times 10^{-10}}$

Solution:

50,000,000

Exercise:

Problem: $\frac{7 \times 10^{-2}}{1 \times 10^{-8}}$

Basic Properties of Real Numbers: The Real Number Line and the Real Numbers

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in this chapter. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret. Objectives of this module: be familiar with the real number line and the real numbers, understand the ordering of the real numbers.

Overview

- The Real Number Line
- The Real Numbers
- Ordering the Real Numbers

The Real Number Line

Real Number Line
In our study of algebra, we will use several collections of numbers. The **real number line** allows us to **visually** display the numbers in which we are interested.

A line is composed of infinitely many points. To each point we can associate a unique number, and with each number we can associate a particular point.

Coordinate
The number associated with a point on the number line is called the **coordinate** of the point.

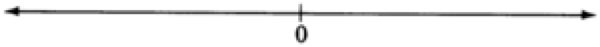
Graph
The point on a line that is associated with a particular number is called the **graph** of that number.

We construct the real number line as follows:

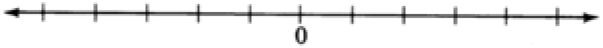
Construction of the Real Number Line

1. Draw a horizontal line.

2. Choose any point on the line and label it 0. This point is called the **origin**.



3. Choose a convenient length. This length is called "1 unit." Starting at 0, mark this length off in both directions, being careful to have the lengths look like they are about the same.



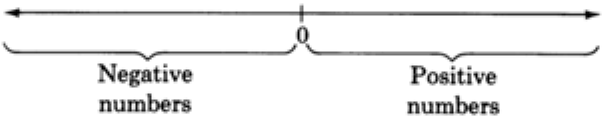
We now define a real number.

Real Number

A **real number** is any number that is the coordinate of a point on the real number line.

Positive and Negative Real Numbers

The collection of these infinitely many numbers is called the **collection of real numbers**. The real numbers whose graphs are to the right of 0 are called the **positive real numbers**. The real numbers whose graphs appear to the left of 0 are called the **negative real numbers**.



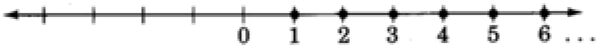
The number 0 is neither positive nor negative.

The Real Numbers

The collection of real numbers has many subcollections. The subcollections that are of most interest to us are listed below along with their notations and graphs.

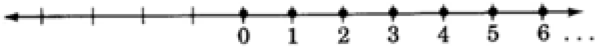
Natural Numbers

The **natural numbers** (N)**:** $\{1,\ 2, 3,\ \ldots\}$



Whole Numbers

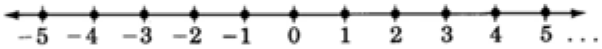
The **whole numbers** (W)**:** $\{0,\ 1,\ 2, 3,\ \ldots\}$



Notice that every natural number is a whole number.

Integers

The **integers** (Z)**:** $\{\ldots,\ -3, -2, -1, 0, 1,\ 2,\ 3,\ \ldots\}$



Notice that every whole number is an integer.

Rational Numbers

The **rational numbers** (Q)**:** Rational numbers are real numbers that can be written in the form a/b, where a and b are integers, and $b \neq 0$.

Fractions

Rational numbers are commonly called **fractions.**

Division by 1

Since b can equal 1, every integer is a rational number: $\frac{a}{1} = a$.

Division by 0

Recall that $10/2 = 5$ since $2 \cdot 5 = 10$. However, if $10/0 = x$, then $0 \cdot x = 10$. But $0 \cdot x = 0$, not 10. This suggests that no quotient exists.

Now consider $0/0 = x$. If $0/0 = x$, then $0 \cdot x = 0$. But this means that x could be any number, that is, $0/0 = 4$ since $0 \cdot 4 = 0$, or $0/0 = 28$ since $0 \cdot 28 = 0$. This suggests that the quotient is indeterminant.

$x/0$ Is Undefined or Indeterminant

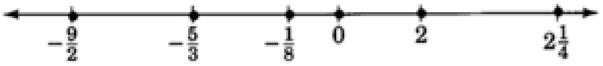
Division by 0 is undefined or indeterminant.

Do not divide by 0.

Rational numbers have decimal representations that either terminate or do not terminate but contain a repeating block of digits. Some examples are:

$$\underset{\text{Terminating}}{\frac{3}{4} = 0.75} \qquad \underset{\text{Nonterminating, but repeating}}{\frac{15}{11} = 1.36363636\ldots}$$

Some rational numbers are graphed below.



Irrational Numbers

The **irrational numbers** (Ir)**:** Irrational numbers are numbers that cannot be written as the quotient of two integers. They are numbers whose decimal representations are nonterminating and nonrepeating. Some examples are

$$4.01001000100001\ldots \qquad \pi = 3.1415927\ldots$$

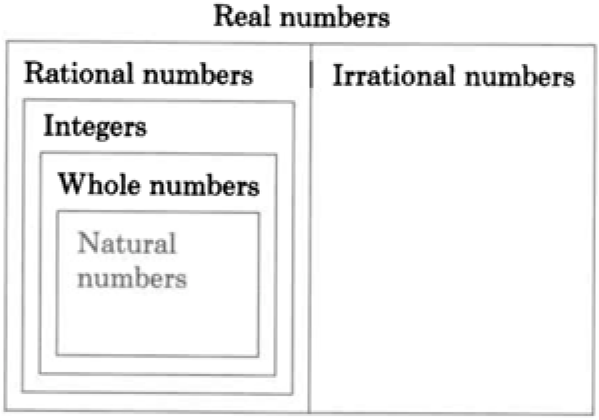
Notice that the collections of rational numbers and irrational numbers have no numbers in common.

When graphed on the number line, the rational and irrational numbers account for every point on the number line. Thus each point on the number line has a coordinate that is either a rational or an irrational number.

In summary, we have

Sample Set A

The summaray chart illustrates that



Example:
Every natural number is a real number.

Example:
Every whole number is a real number.

Example:
No integer is an irrational number.

Practice Set A

Exercise:

Problem: Is every natural number a whole number?

Solution:

yes

Exercise:

Problem: Is every whole number an integer?

Solution:

yes

Exercise:

Problem: Is every integer a rational number?

Solution:

yes

Exercise:

Problem: Is every rational number a real number?

Solution:

yes

Exercise:

Problem: Is every integer a natural number?

Solution:

no

Exercise:

Problem: Is there an integer that is a natural number?

Solution:

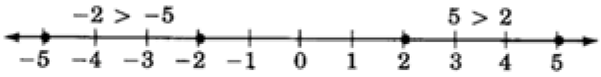
yes

Ordering the Real Numbers

Ordering the Real Numbers
A real number b is said to be greater than a real number a, denoted $b > a$, if the graph of b is to the right of the graph of a on the number line.

Sample Set B

As we would expect, $5 > 2$ since 5 is to the right of 2 on the number line. Also, $-2 > -5$ since -2 is to the right of -5 on the number line.



Practice Set B

Exercise:

Problem: Are all positive numbers greater than 0?

Solution:

yes

Exercise:

Problem: Are all positive numbers greater than all negative numbers?

Solution:

yes

Exercise:

Problem: Is 0 greater than all negative numbers?

Solution:

yes

Exercise:

Problem:

Is there a largest positive number? Is there a smallest negative number?

Solution:

no, no

Exercise:

Problem:

How many real numbers are there? How many real numbers are there between 0 and 1?

Solution:

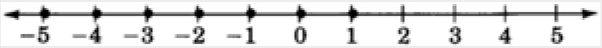
infinitely many, infinitely many

Sample Set C

Example:
What integers can replace x so that the following statement is true?

$$-4 \leq x < 2$$

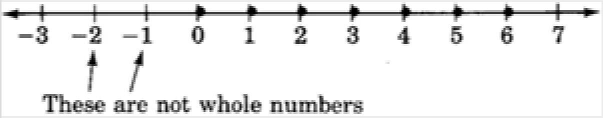
This statement indicates that the number represented by x is between -4 and 2. Specifically, -4 is less than or equal to x, and at the same time, x is strictly less than 2. This statement is an example of a compound inequality.



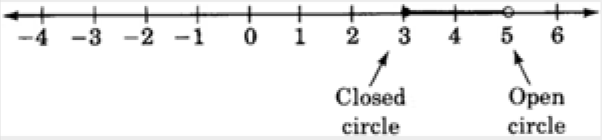
The integers are $-4, \ -3, \ -2, \ -1, \ 0, \ 1.$

Example:
Draw a number line that extends from -3 to 7. Place points at all whole numbers between and including -2 and 6.

Example:
Draw a number line that extends from -4 to 6 and place points at all real numbers greater than or equal to 3 but strictly less than 5.



It is customary to use a **closed circle** to indicate that a point is included in the graph and an **open circle** to indicate that a point is not included.



Practice Set C

Exercise:

Problem:

What whole numbers can replace x so that the following statement is true?

$-3 \leq x < 3$

Solution:

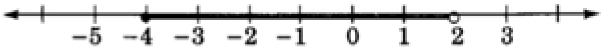
0, 1, 2

Exercise:

Problem:

Draw a number line that extends from -5 to 3 and place points at all numbers greater than or equal to -4 but strictly less than 2.

Solution:



Exercises

For the following problems, next to each real number, note all collections to which it belongs by writing N for natural numbers, W for whole numbers, Z for integers, Q for rational numbers, Ir for irrational numbers, and R for real numbers. Some numbers may require more than one letter.

Exercise:

Problem: $\frac{1}{2}$

Solution:

$Q,\ R$

Exercise:

Problem: -12

Exercise:

Problem: 0

Solution:

$W,\ Z,\ Q,\ R$

Exercise:

Problem: $-24\frac{7}{8}$

Exercise:

Problem: $86.3333\ldots$

Solution:

$Q,\ R$

Exercise:

Problem: $49.125125125\ldots$

Exercise:

Problem: -15.07

Solution:

$Q,\ R$

For the following problems, draw a number line that extends from -3 to 3. Locate each real number on the number line by placing a point (closed circle) at its approximate location.

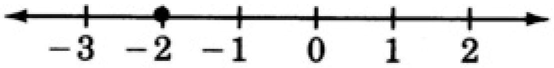
Exercise:

Problem: $1\frac{1}{2}$

Exercise:

Problem: -2

Solution:



Exercise:

Problem: $-\frac{1}{8}$

Exercise:

Problem: Is 0 a positive number, negative number, neither, or both?

Solution:

neither

Exercise:

Problem:

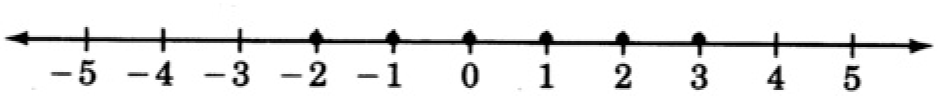
An integer is an even integer if it can be divided by 2 without a remainder; otherwise the number is odd. Draw a number line that extends from -5 to 5 and place points at all negative even integers and at all positive odd integers.

Exercise:

Problem:

Draw a number line that extends from -5 to 5. Place points at all integers strictly greater than -3 but strictly less than 4.

Solution:



For the following problems, draw a number line that extends from -5 to 5. Place points at all real numbers between and including each pair of numbers.

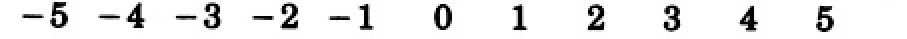
Exercise:

Problem: -5 and -2

Exercise:

Problem: -3 and 4

Solution:



Exercise:

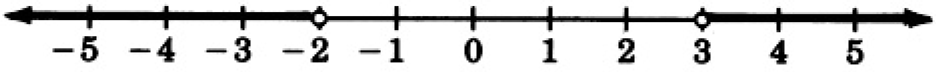
Problem: -4 and 0

Exercise:

Problem:

Draw a number line that extends from -5 to 5. Is it possible to locate any numbers that are strictly greater than 3 but also strictly less than -2?

Solution:



; no

For the pairs of real numbers shown in the following problems, write the appropriate relation symbol $(<,\ >,\ =)$ in place of the $*$.

Exercise:

Problem: $-5 * -1$

Exercise:

Problem: $-3 * 0$

Solution:

$<$

Exercise:

Problem: $-4 * 7$

Exercise:

Problem: $6 * -1$

Solution:

$>$

Exercise:

Problem: $-\frac{1}{4} * -\frac{3}{4}$

Exercise:

Problem: Is there a largest real number? If so, what is it?

Solution:

no

Exercise:

Problem: Is there a largest integer? If so, what is it?

Exercise:

Problem: Is there a largest two-digit integer? If so, what is it?

Solution:

99

Exercise:

Problem: Is there a smallest integer? If so, what is it?

Exercise:

Problem: Is there a smallest whole number? If so, what is it?

Solution:

yes, 0

For the following problems, what numbers can replace x so that the following statements are true?

Exercise:

Problem: $-1 \le x \le 5 \quad x\,\text{an integer}$

Exercise:

Problem: $-7 < x < -1, \quad x\,\text{an integer}$

Solution:

$-6, \; -5, \; -4, \; -3, \; -2$

Exercise:

Problem: $-3 \le x \le 2, \quad x\,\text{a natural number}$

Exercise:

Problem: $-15 < x \le -1, \quad x\,\text{a natural number}$

Solution:

There are no natural numbers between −15 and −1.

Exercise:

Problem: $-5 \le x < 5, \quad x\,\text{a whole number}$

Exercise:

Problem:

The temperature in the desert today was ninety-five degrees. Represent this temperature by a rational number.

Solution:

$\left(\frac{95}{1}\right)^{\circ}$

Exercise:

Problem:

The temperature today in Colorado Springs was eight degrees below zero. Represent this temperature with a real number.

Exercise:

Problem: Is every integer a rational number?

Solution:

Yes, every integer is a rational number.

Exercise:

Problem: Is every rational number an integer?

Exercise:

Problem:

Can two rational numbers be added together to yield an integer? If so, give an example.

Solution:

Yes. $\frac{1}{2} + \frac{1}{2} = 1 \text{or} 1 + 1 = 2$

For the following problems, on the number line, how many units (intervals) are there between?

Exercise:

Problem: 0 and 2?

Exercise:

Problem: -5 and 0?

Solution:

5 units

Exercise:

Problem: 0 and 6?

Exercise:

Problem: -8 and 0?

Solution:

8 units

Exercise:

Problem: -3 and 4?

Exercise:

Problem: m and n, $m > n$?

Solution:

$m - n$ units

Exercise:

Problem: $-a$ and $-b$, $-b > -a$?

Exercises for Review

Exercise:

Problem: (**[link]**) Find the value of $6 + 3(15 - 8) - 4$.

Solution:

23

Exercise:

Problem: (**[link]**) Find the value of $5(8 - 6) + 3(5 + 2 \cdot 3)$.

Exercise:

Problem:

(**[link]**) Are the statements $y < 4$ and $y \geq 4$ the same or different?

Solution:

different

Exercise:

Problem:

(**[link]**) Use algebraic notation to write the statement "six times a number is less than or equal to eleven."

Exercise:

Problem:

(**[link]**) Is the statement $8(15 - 3 \cdot 4) - 3 \cdot 7 \geq 3$ true or false?

Solution:

true

Use the Rectangular Coordinate System
By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

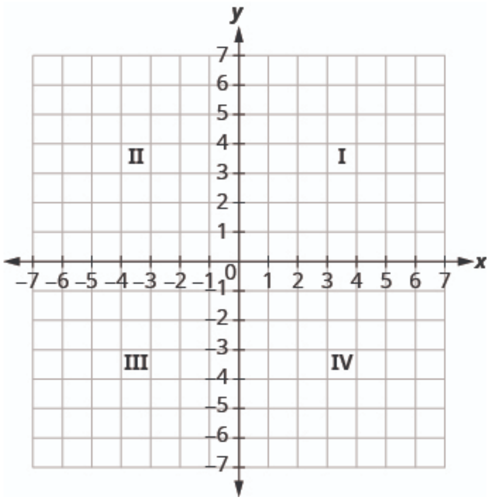
Note:
Before you get started, take this readiness quiz.

1. Evaluate $x+3$ when $x=-1$.
   If you missed this problem, review [link].
2. Evaluate $2x-5y$ when $x=3$ and $y=-2$.
   If you missed this problem, review [link].
3. Solve for y: $40-4y=20$.
   If you missed this problem, review [link].

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the ‘coordinate plane’.

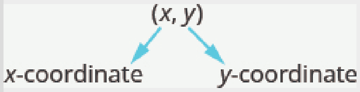
The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis*. The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See [link].



‘Quadrant’ has the root ‘quad,’ which means ‘four.’

In the rectangular coordinate system, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

Note:
Ordered Pair
An **ordered pair**, (x, y), gives the coordinates of a point in a rectangular coordinate system.

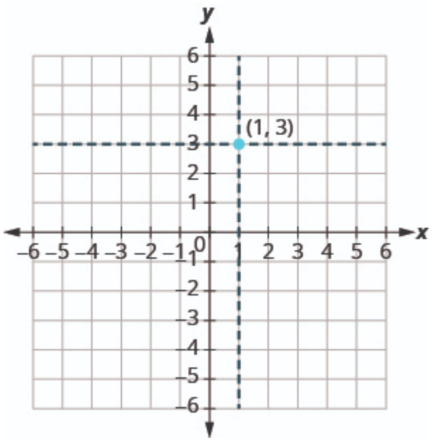


The first number is the *x*-coordinate.
The second number is the *y*-coordinate.

The phrase ‘ordered pair’ means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the origin.

Note:
The Origin
The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let’s plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$.



Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example:
Exercise:

Problem:

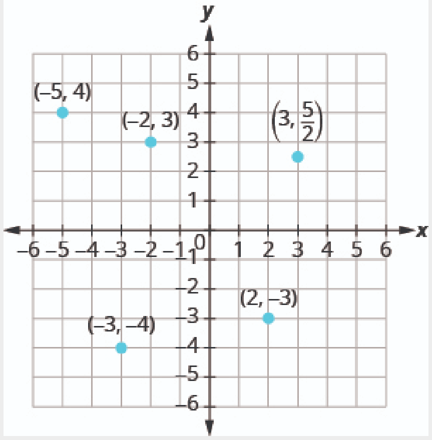
Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located:

ⓐ $(-5,4)$ ⓑ $(-3,-4)$ ⓒ $(2,-3)$ ⓓ $(-2,3)$ ⓔ $\left(3,\frac{5}{2}\right)$.

Solution:
Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

ⓐ Since $x=-5$, the point is to the left of the *y*-axis. Also, since $y=4$, the point is above the *x*-axis. The point $(-5,4)$ is in Quadrant II.
ⓑ Since $x=-3$, the point is to the left of the *y*-axis. Also, since $y=-4$, the point is below the *x*-axis. The point $(-3,-4)$ is in Quadrant III.
ⓒ Since $x=2$, the point is to the right of the *y*-axis. Since $y=-3$, the point is below the *x*-axis. The point $(2,-3)$ is in Quadrant lV.
ⓓ Since $x=-2$, the point is to the left of the *y*-axis. Since $y=3$, the point is above the *x*-axis. The point $(-2,3)$ is in Quadrant II.
ⓔ Since $x=3$, the point is to the right of the *y*-axis. Since $y=\frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $\left(3,\frac{5}{2}\right)$ is in Quadrant I.



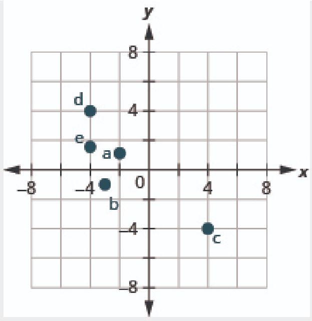
Note:
Exercise:

Problem:

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

ⓐ $(-2,1)$ ⓑ $(-3,-1)$ ⓒ $(4,-4)$ ⓓ $(-4,4)$ ⓔ $\left(-4,\frac{3}{2}\right)$.

Solution:



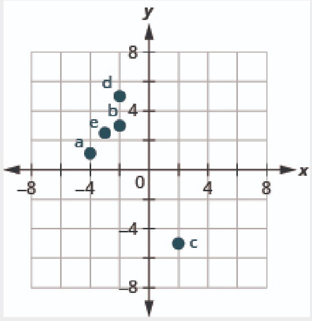
Note:
Exercise:

Problem:

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

ⓐ $(-4, 1)$ ⓑ $(-2, 3)$ ⓒ $(2, -5)$ ⓓ $(-2, 5)$ ⓔ $\left(-3, \frac{5}{2}\right)$.

Solution:



How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

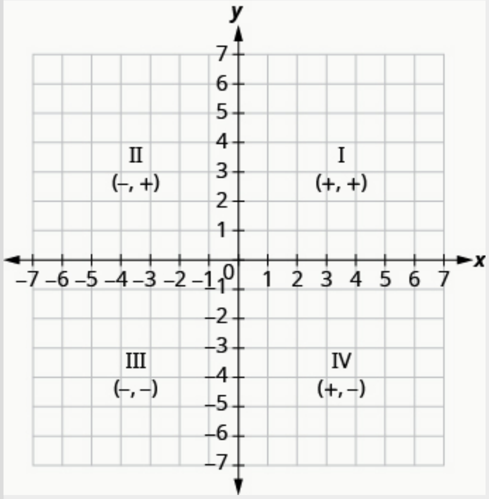
For the point in [link] in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point $(-2, 5)$ is located? In which quadrant is $(2, -5)$ located?

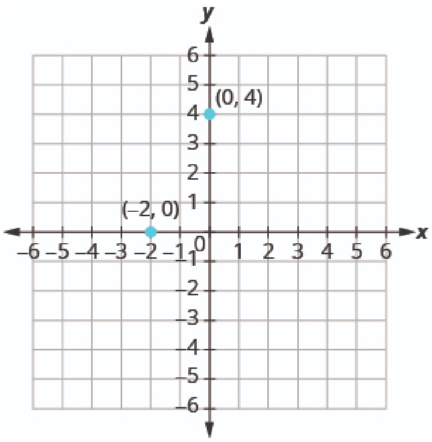
Note:
Quadrants
We can summarize sign patterns of the quadrants in this way.

Equation:

$$\begin{array}{cccc} \text{Quadrant I} & \text{Quadrant II} & \text{Quadrant III} & \text{Quadrant IV} \\ (x,y) & (x,y) & (x,y) & (x,y) \\ (+,+) & (-,+) & (-,-) & (+,-) \end{array}$$



What if one coordinate is zero as shown in [link]? Where is the point $(0,4)$ located? Where is the point $(-2,0)$ located?



The point $(0,4)$ is on the *y*-axis and the point $(-2,0)$ is on the *x*-axis.

Note:
Points on the Axes
Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates $(a,0)$.
Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates $(0,b)$.

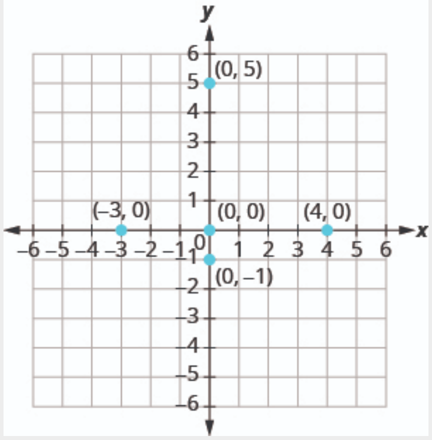
Example:
Exercise:

Problem: Plot each point:

ⓐ $(0,5)$ ⓑ $(4,0)$ ⓒ $(-3,0)$ ⓓ $(0,0)$ ⓔ $(0,-1)$.

Solution:
Solution

ⓐ Since $x=0$, the point whose coordinates are $(0,5)$ is on the *y*-axis.
ⓑ Since $y=0$, the point whose coordinates are $(4,0)$ is on the *x*-axis.
ⓒ Since $y=0$, the point whose coordinates are $(-3,0)$ is on the *x*-axis.
ⓓ Since $x=0$ and $y=0$, the point whose coordinates are $(0,0)$ is the origin.
ⓔ Since $x=0$, the point whose coordinates are $(0,-1)$ is on the *y*-axis.

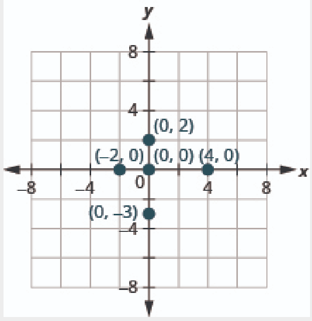


Note:
Exercise:

Problem: Plot each point:

ⓐ $(4,0)$ ⓑ $(-2,0)$ ⓒ $(0,0)$ ⓓ $(0,2)$ ⓔ $(0,-3)$.

Solution:

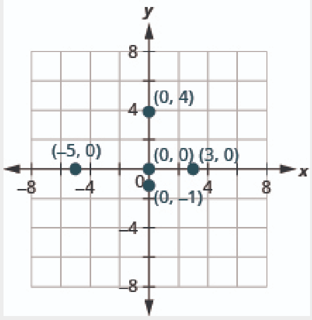


Note:
Exercise:

Problem: Plot each point:

ⓐ $(-5, 0)$ ⓑ $(3, 0)$ ⓒ $(0, 0)$ ⓓ $(0, -1)$ ⓔ $(0, 4)$.

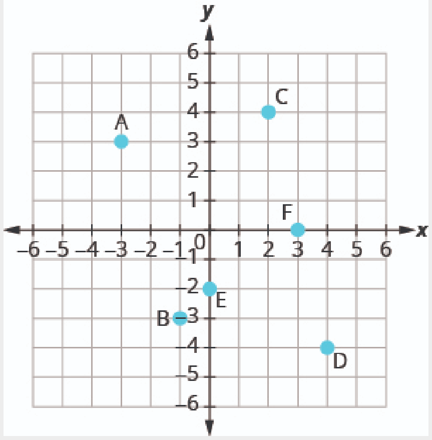
Solution:



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the ordered pair use the correct order, (x, y).

Example:
Exercise:

Problem: Name the ordered pair of each point shown in the rectangular coordinate system.



Solution:
Solution

Point A is above -3 on the *x*-axis, so the *x*-coordinate of the point is -3.

- The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.
- The coordinates of the point are $(-3, 3)$.

Point B is below -1 on the *x*-axis, so the *x*-coordinate of the point is -1.

- The point is to the left of -3 on the *y*-axis, so the *y*-coordinate of the point is -3.
- The coordinates of the point are $(-1, -3)$.

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2.

- The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.
- The coordinates of the point are $(2, 4)$.

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4.

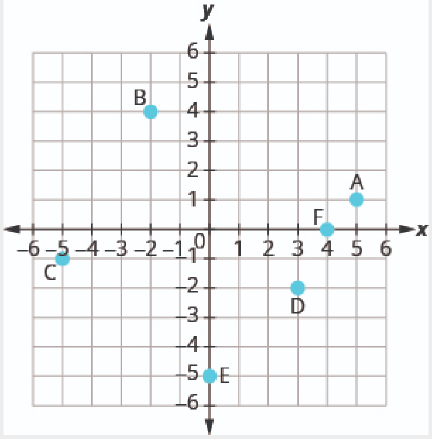
- The point is to the right of -4 on the *y*-axis, so the *y*-coordinate of the point is -4.
- The coordinates of the point are $(4, -4)$.

Point E is on the *y*-axis at $y = -2$. The coordinates of point E are $(0, -2)$.

Point F is on the *x*-axis at $x = 3$. The coordinates of point F are $(3, 0)$.

Note:
Exercise:

Problem: Name the ordered pair of each point shown in the rectangular coordinate system.

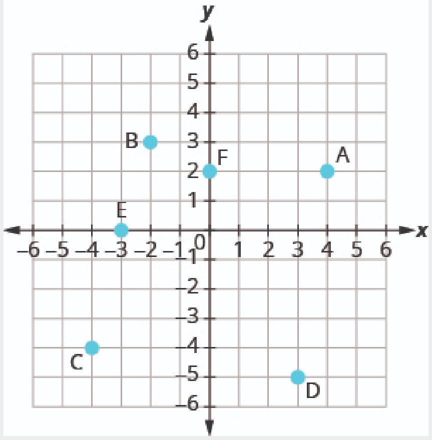


Solution:

A: $(5, 1)$ B: $(-2, 4)$ C: $(-5, -1)$ D: $(3, -2)$ E: $(0, -5)$ F: $(4, 0)$

Note:
Exercise:

Problem: Name the ordered pair of each point shown in the rectangular coordinate system.



Solution:

A: $(4,2)$ B: $(-2,3)$ C: $(-4,-4)$ D: $(3,-5)$ E: $(-3,0)$ F: $(0,2)$

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like $x=4$. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.
Equation:

$$\begin{aligned} 3x+5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

But equations can have more than one variable. Equations with two variables may be of the form $Ax+By=C$. Equations of this form are called **linear equations in two variables**.

Note:
Linear Equation
An equation of the form $Ax+By=C$, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$\begin{aligned} Ax + By &= C \\ x + 4y &= 8 \\ A = 1,\ B = 4,\ C &= 8 \end{aligned}$$

The equation $y = -3x + 5$ is also a linear equation. But it does not appear to be in the form $Ax + By = C$. We can use the Addition Property of Equality and rewrite it in $Ax + By = C$ form.

	$y = -3x + 5$
Add to both sides.	$y + 3x = -3x + 5 + 3x$
Simplify.	$y + 3x = 5$
Use the Commutative Property to put it in $Ax + By = C$ form.	$3x + y = 5$

By rewriting $y = -3x + 5$ as $3x + y = 5$, we can easily see that it is a linear equation in two variables because it is of the form $Ax + By = C$. When an equation is in the form $Ax + By = C$, we say it is in *standard form*.

Note:
Standard Form of Linear Equation
A linear equation is in standard form when it is written $Ax + By = C$.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x, y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Note:
Solution of a Linear Equation in Two Variables
An **ordered pair** (x, y) is a **solution** of the linear equation $Ax + By = C$, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

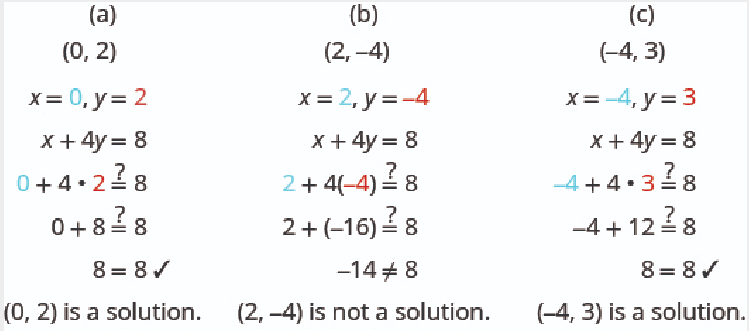
Example:
Exercise:

Problem: Determine which ordered pairs are solutions to the equation $x + 4y = 8$.

ⓐ $(0, 2)$ ⓑ $(2, -4)$ ⓒ $(-4, 3)$

Solution:
Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.



(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \overset{?}{=} 8$	$2 + 4(-4) \overset{?}{=} 8$	$-4 + 4 \cdot 3 \overset{?}{=} 8$
$0 + 8 \overset{?}{=} 8$	$2 + (-16) \overset{?}{=} 8$	$-4 + 12 \overset{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Note:
Exercise:

Problem: Which of the following ordered pairs are solutions to $2x + 3y = 6$?

ⓐ $(3, 0)$ ⓑ $(2, 0)$ ⓒ $(6, -2)$

Solution:

a, c

Note:
Exercise:

Problem: Which of the following ordered pairs are solutions to the equation $4x - y = 8$?

ⓐ $(0, 8)$ ⓑ $(2, 0)$ ⓒ $(1, -4)$

Solution:

b, c

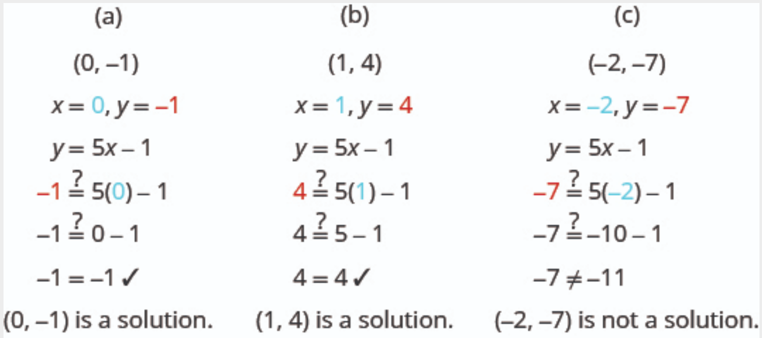
Example:
Exercise:

Problem: Which of the following ordered pairs are solutions to the equation $y = 5x - 1$?

ⓐ $(0, -1)$ ⓑ $(1, 4)$ ⓒ $(-2, -7)$

Solution:
Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if it results in a true statement.



(a)	(b)	(c)
$(0, -1)$	$(1, 4)$	$(-2, -7)$
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \overset{?}{=} 5(0) - 1$	$4 \overset{?}{=} 5(1) - 1$	$-7 \overset{?}{=} 5(-2) - 1$
$-1 \overset{?}{=} 0 - 1$	$4 \overset{?}{=} 5 - 1$	$-7 \overset{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
$(0, -1)$ is a solution.	$(1, 4)$ is a solution.	$(-2, -7)$ is not a solution.

Note:
Exercise:

Problem: Which of the following ordered pairs are solutions to the equation $y = 4x - 3$?

ⓐ $(0, 3)$ ⓑ $(1, 1)$ ⓒ $(-1, -1)$

Solution:

b

Note:
Exercise:

Problem: Which of the following ordered pairs are solutions to the equation $y = -2x + 6$?

ⓐ $(0, 6)$ ⓑ $(1, 4)$ ⓒ $(-2, -2)$

Solution:

a, b

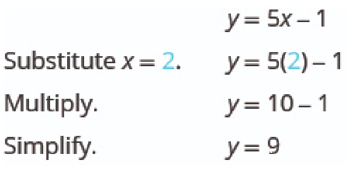
Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*- and *y*-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation $y = 5x - 1$ that we found in [link]. We can summarize this information in a table of solutions, as shown in [link].

$y=5x-1$		
x	y	(x,y)
0	-1	$(0,-1)$
1	4	$(1,4)$

To find a third solution, we’ll let $x=2$ and solve for y.



	$y=5x-1$
Substitute $x=2$.	$y=5(2)-1$
Multiply.	$y=10-1$
Simplify.	$y=9$

The ordered pair $(2,9)$ is a solution to $y=5x-1$. We will add it to [link].

$y=5x-1$		
x	y	(x,y)
0	-1	$(0,-1)$
1	4	$(1,4)$
2	9	$(2,9)$

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Example:
Exercise:

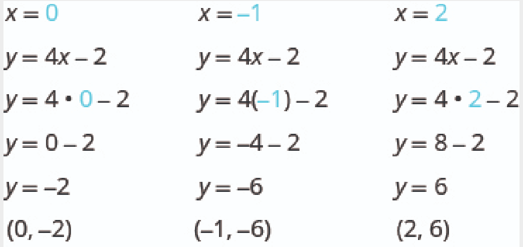
Problem: Complete [link] to find three solutions to the equation $y=4x-2$.

$y=4x-2$		
x	y	(x,y)

0		
-1		
2		

Solution:
Solution

Substitute $x=0$, $x=-1$, and $x=2$ into $y=4x-2$.



$x=0$	$x=-1$	$x=2$
$y=4x-2$	$y=4x-2$	$y=4x-2$
$y=4\cdot 0-2$	$y=4(-1)-2$	$y=4\cdot 2-2$
$y=0-2$	$y=-4-2$	$y=8-2$
$y=-2$	$y=-6$	$y=6$
$(0,-2)$	$(-1,-6)$	$(2,6)$

The results are summarized in [link].

$y=4x-2$		
x	y	(x,y)
0	-2	$(0,-2)$
-1	-6	$(-1,-6)$
2	6	$(2,6)$

Note:
Exercise:

Problem: Complete the table to find three solutions to this equation: $y=3x-1$.

$y=3x-1$

x	y	(x, y)
0		
-1		
2		

Solution:

$y = 3x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
-1	-4	$(-1, -4)$
2	5	$(2, 5)$

Note:
Exercise:

Problem: Complete the table to find three solutions to this equation: $y = 6x + 1$.

$y = 6x + 1$		
x	y	(x, y)
0		
1		
-2		

Solution:

$y = 6x + 1$		
x	y	(x, y)
0	1	$(0, 1)$
1	7	$(1, 7)$
-2	-11	$(-2, -11)$

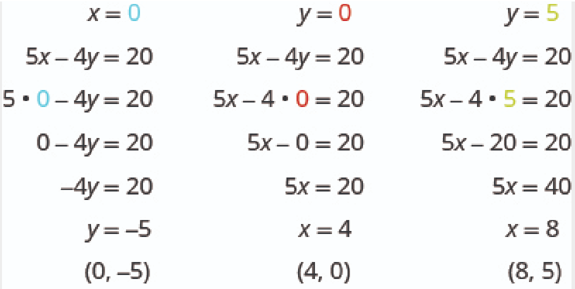
Example:
Exercise:

Problem: Complete [link] to find three solutions to the equation $5x - 4y = 20$.

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution:
Solution

Substitute the given value into the equation $5x - 4y = 20$ and solve for the other variable. Then, fill in the values in the table.



$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in [link].

$5x - 4y = 20$		
x	y	(x, y)
0	-5	$(0, -5)$
4	0	$(4, 0)$
8	5	$(8, 5)$

Note:
Exercise:

Problem: Complete the table to find three solutions to this equation: $2x - 5y = 20$.

$2x - 5y = 20$		
x	y	(x, y)
0		
	0	
-5		

Solution:

$2x-5y=20$		
x	y	(x,y)
0	-4	$(0,-4)$
10	0	$(10,0)$
-5	-6	$(-5,-6)$

Note:
Exercise:

Problem: Complete the table to find three solutions to this equation: $3x-4y=12$.

$3x-4y=12$		
x	y	(x,y)
0		
	0	
-4		

Solution:

$3x-4y=12$		
x	y	(x,y)
0	-3	$(0,-3)$
4	0	$(4,0)$
-4	-6	$(-4,-6)$

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example:
Exercise:

Problem: Find three solutions to the equation $y = -3x + 2$.

Solution:
Solution

We can substitute any value we want for x or any value for y. Since the equation is in *y*-form, it will be easier to substitute in values of x. Let's pick $x = 0$, $x = 1$, and $x = -1$.

			$x = 0$	$x = 1$	$x = -1$
			$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$
Substitute the value into the equation.			$y = -3 \cdot 0 + 2$	$y = -3 \cdot 1 + 2$	$y = -3(-1) + 2$
Simplify.			$y = 0 + 2$	$y = -3 + 2$	$y = 3 + 2$
Simplify.			$y = 2$	$y = -1$	$y = 5$
Write the ordered pair.			(0, 2)	(1, −1)	(−1, 5)
Check.					
$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$			

$2 \overset{?}{=} -3 \cdot 0 + 2$	$-1 \overset{?}{=} -3 \cdot 1 + 2$	$5 \overset{?}{=} -3(-1) + 2$			
$2 \overset{?}{=} 0 + 2$	$-1 \overset{?}{=} -3 + 2$	$5 \overset{?}{=} 3 + 2$			
$2 = 2\checkmark$	$-1 = -1\checkmark$	$5 = 5\checkmark$			

So, $(0, 2)$, $(1, -1)$ and $(-1, 5)$ are all solutions to $y = -3x + 2$. We show them in [link].

$y = -3x + 2$		
x	y	(x, y)
0	2	$(0, 2)$
1	-1	$(1, -1)$
-1	5	$(-1, 5)$

Note:
Exercise:

Problem: Find three solutions to this equation: $y = -2x + 3$.

Solution:

Answers will vary.

Note:
Exercise:

Problem: Find three solutions to this equation: $y = -4x + 1$.

Solution:

Answers will vary.

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when $x = 0$ find a second solution when $y = 0$, and then find a third solution.

Example:
Exercise:

Problem: Find three solutions to the equation $3x+2y=6$.

Solution:
Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first $x=0$, then $y=0$, and then find a third point.

	$x=0$	$y=0$	$x=1$
	$3x+2y=6$	$3x+2y=6$	$3x+2y=6$
Substitute the value into the equation.	$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.	$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.	$2y=6$	$3x=6$	$2y=3$
	$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.	(0, 3)	(2, 0)	$\left(1,\frac{3}{2}\right)$
Check.			

$3x+2y=6$	$3x+2y=6$	$3x+2y=6$			
$3\cdot 0+2\cdot 3\overset{?}{=}6$	$3\cdot 2+2\cdot 0\overset{?}{=}6$	$3\cdot 1+2\cdot\frac{3}{2}\overset{?}{=}6$			
$0+6\overset{?}{=}6$	$6+0\overset{?}{=}6$	$3+3\overset{?}{=}6$			
$6=6\checkmark$	$6=6\checkmark$	$6=6\checkmark$			

So $(0,3)$, $(2,0)$, and $\left(1,\frac{3}{2}\right)$ are all solutions to the equation $3x+2y=6$. We can list these three solutions in [link].

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$\left(1, \frac{3}{2}\right)$

Note:
Exercise:

Problem: Find three solutions to the equation $2x + 3y = 6$.

Solution:

Answers will vary.

Note:
Exercise:

Problem: Find three solutions to the equation $4x + 2y = 8$.

Solution:

Answers will vary.

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
  - On the *x*-axis, $y = 0$. Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates $(a, 0)$.
  - On the *y*-axis, $x = 0$. Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates $(0, b)$.

- **Solution of a Linear Equation**

- An ordered pair (x, y) is a solution of the linear equation $Ax + By = C$, if the equation is a true statement when the *x*- and *y*- values of the ordered pair are substituted into the equation.

Practice Makes Perfect

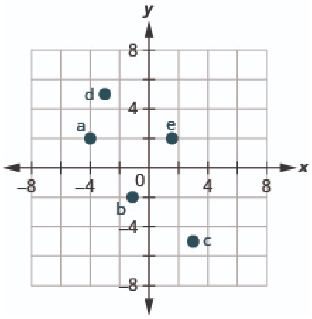
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

Exercise:

ⓐ $(-4, 2)$
ⓑ $(-1, -2)$
ⓒ $(3, -5)$
ⓓ $(-3, 5)$
Problem: ⓔ $\left(\frac{5}{3}, 2\right)$

Solution:



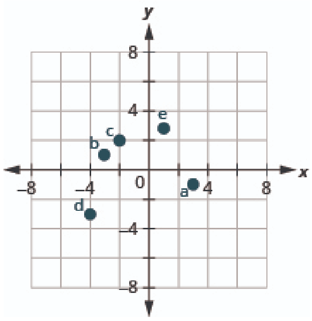
Exercise:

ⓐ $(-2, -3)$
ⓑ $(3, -3)$
ⓒ $(-4, 1)$
ⓓ $(4, -1)$
Problem: ⓔ $\left(\frac{3}{2}, 1\right)$

Exercise:

ⓐ $(3, -1)$
ⓑ $(-3, 1)$
ⓒ $(-2, 2)$
ⓓ $(-4, -3)$
Problem: ⓔ $\left(1, \frac{14}{5}\right)$

Solution:



Exercise:

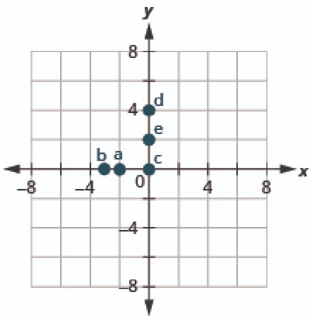
ⓐ $(-1, 1)$
ⓑ $(-2, -1)$
ⓒ $(2, 1)$
ⓓ $(1, -4)$
Problem: ⓔ $\left(3, \frac{7}{2}\right)$

In the following exercises, plot each point in a rectangular coordinate system.

Exercise:

ⓐ $(-2, 0)$
ⓑ $(-3, 0)$
ⓒ $(0, 0)$
ⓓ $(0, 4)$
Problem: ⓔ $(0, 2)$

Solution:



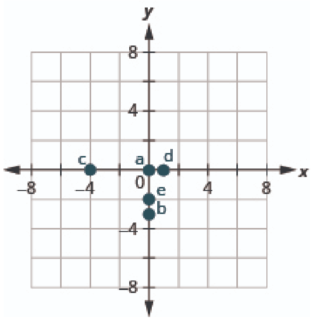
Exercise:

ⓐ $(0, 1)$
ⓑ $(0, -4)$
ⓒ $(-1, 0)$
ⓓ $(0, 0)$
Problem: ⓔ $(5, 0)$

Exercise:

ⓐ $(0, 0)$
ⓑ $(0, -3)$
ⓒ $(-4, 0)$
ⓓ $(1, 0)$
Problem: ⓔ $(0, -2)$

Solution:



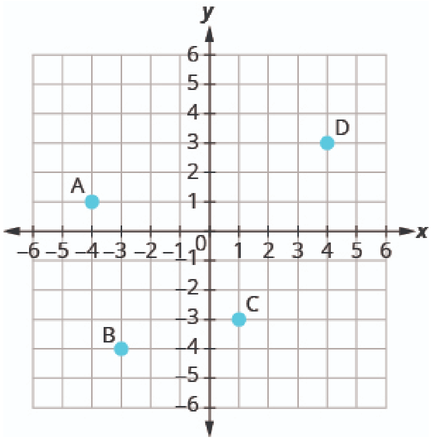
Exercise:

ⓐ $(-3, 0)$
ⓑ $(0, 5)$
ⓒ $(0, -2)$
ⓓ $(2, 0)$
Problem: ⓔ $(0, 0)$

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise:

Problem:

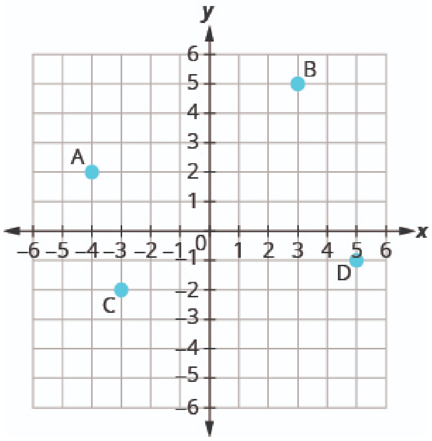


Solution:

A: $(-4, 1)$ B: $(-3, -4)$ C: $(1, -3)$ D: $(4, 3)$

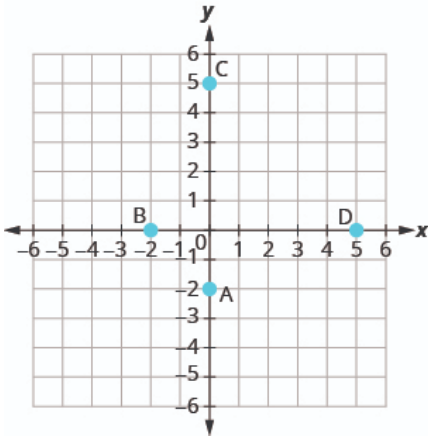
Exercise:

Problem:



Exercise:

Problem:

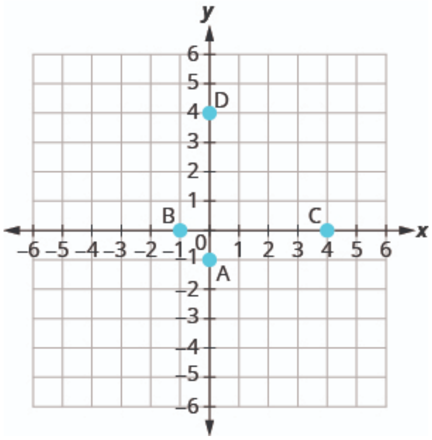


Solution:

A: $(0, -2)$ B: $(-2, 0)$ C: $(0, 5)$ D: $(5, 0)$

Exercise:

Problem:



Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise:

Problem: $2x + y = 6$

ⓐ $(1, 4)$
ⓑ $(3, 0)$
ⓒ $(2, 3)$

Solution:

a, b

Exercise:

Problem: $x + 3y = 9$

ⓐ $(0, 3)$
ⓑ $(6, 1)$
ⓒ $(-3, -3)$

Exercise:

Problem: $4x - 2y = 8$

ⓐ $(3, 2)$
ⓑ $(1, 4)$
ⓒ $(0, -4)$

Solution:

a, c

Exercise:

Problem: $3x - 2y = 12$

ⓐ $(4, 0)$
ⓑ $(2, -3)$
ⓒ $(1, 6)$

Exercise:

Problem: $y = 4x + 3$

ⓐ $(4, 3)$
ⓑ $(-1, -1)$
ⓒ $\left(\frac{1}{2}, 5\right)$

Solution:

b, c

Exercise:

Problem: $y = 2x - 5$

ⓐ $(0, -5)$
ⓑ $(2, 1)$
ⓒ $\left(\frac{1}{2}, -4\right)$

Exercise:

Problem: $y = \frac{1}{2}x - 1$

ⓐ $(2, 0)$
ⓑ $(-6, -4)$
ⓒ $(-4, -1)$

Solution:

a, b

Exercise:

Problem: $y = \frac{1}{3}x + 1$

ⓐ $(-3, 0)$
ⓑ $(9, 4)$
ⓒ $(-6, -1)$

Complete a Table of Solutions to a Linear Equation

In the following exercises, complete the table to find solutions to each linear equation.

Exercise:

Problem: $y = 2x - 4$

x	y	(x, y)
0		
2		
-1		

Solution:

x	y	(x, y)
0	-4	$(0, -4)$
2	0	$(2, 0)$
-1	-6	$(-1, -6)$

Exercise:

Problem: $y = 3x - 1$

x	y	(x, y)
0		
2		
-1		

Exercise:

Problem: $y = -x + 5$

x	y	(x, y)
0		
3		
-2		

Solution:

x	y	(x, y)
0	5	$(0, 5)$
3	2	$(3, 2)$
-2	7	$(-2, 7)$

Exercise:

Problem: $y = -x + 2$

x	y	(x, y)
0		
3		
-2		

Exercise:

Problem: $y = \frac{1}{3}x + 1$

x	y	(x, y)
0		
3		
6		

Solution:

x	y	(x, y)
0	1	$(0, 1)$
3	2	$(3, 2)$
6	3	$(6, 3)$

Exercise:

Problem: $y = \frac{1}{2}x + 4$

x	y	(x, y)
0		
2		
4		

Exercise:

Problem: $y = -\frac{3}{2}x - 2$

x	y	(x, y)
0		
2		
-2		

Solution:

x	y	(x, y)
0	-2	$(0, -2)$
2	-5	$(2, -5)$
-2	1	$(-2, 1)$

Exercise:

Problem: $y = -\frac{2}{3}x - 1$

x	y	(x, y)
0		
3		
-3		

Exercise:

Problem: $x + 3y = 6$

x	y	(x, y)
0		
3		
	0	

Solution:

x	y	(x, y)
0	2	$(0, 2)$
3	4	$(3, 1)$
6	0	$(6, 0)$

Exercise:

Problem: $x + 2y = 8$

x	y	(x, y)
0		
4		
	0	

Exercise:

Problem: $2x - 5y = 10$

x	y	(x,y)
0		
10		
	0	

Solution:

x	y	(x,y)
0	-2	$(0,-2)$
10	2	$(10,2)$
5	0	$(5,0)$

Exercise:

Problem: $3x-4y=12$

x	y	(x,y)
0		
8		
	0	

Find Solutions to a Linear Equation

In the following exercises, find three solutions to each linear equation.

Exercise:

Problem: $y=5x-8$

Solution:

Answers will vary.

Exercise:

Problem: $y = 3x - 9$

Exercise:

Problem: $y = -4x + 5$

Solution:

Answers will vary.

Exercise:

Problem: $y = -2x + 7$

Exercise:

Problem: $x + y = 8$

Solution:

Answers will vary.

Exercise:

Problem: $x + y = 6$

Exercise:

Problem: $x + y = -2$

Solution:

Answers will vary.

Exercise:

Problem: $x + y = -1$

Exercise:

Problem: $3x + y = 5$

Solution:

Answers will vary.

Exercise:

Problem: $2x + y = 3$

Exercise:

Problem: $4x - y = 8$

Solution:

Answers will vary.

Exercise:

Problem: $5x - y = 10$

Exercise:

Problem: $2x + 4y = 8$

Solution:

Answers will vary.

Exercise:

Problem: $3x + 2y = 6$

Exercise:

Problem: $5x - 2y = 10$

Solution:

Answers will vary.

Exercise:

Problem: $4x - 3y = 12$

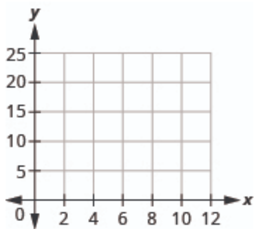
Everyday Math

Exercise:

Problem:

Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

ⓐ Plot the points on a coordinate plane.

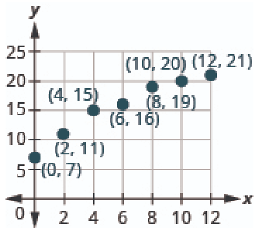


ⓑ Why is only Quadrant I needed?

Age x	**Weight** y	(x, y)
0	7	(0, 7)
2	11	(2, 11)
4	15	(4, 15)
6	16	(6, 16)
8	19	(8, 19)
10	20	(10, 20)
12	21	(12, 21)

Solution:

ⓐ



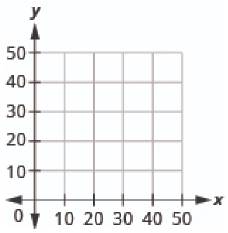
ⓑ Age and weight are only positive.

Exercise:

Problem:

Weight of a child. Latresha recorded her son’s height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

ⓐ Plot the points on a coordinate plane.



ⓑ Why is only Quadrant I needed?

Height x	**Weight** y	(x, y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)
37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

Writing Exercises

Exercise:

Problem: Explain in words how you plot the point $(4, -2)$ in a rectangular coordinate system.

Solution:

Answers will vary.

Exercise:

Problem: How do you determine if an ordered pair is a solution to a given equation?

Exercise:

Problem: Is the point $(-3, 0)$ on the *x*-axis or *y*-axis? How do you know?

Solution:

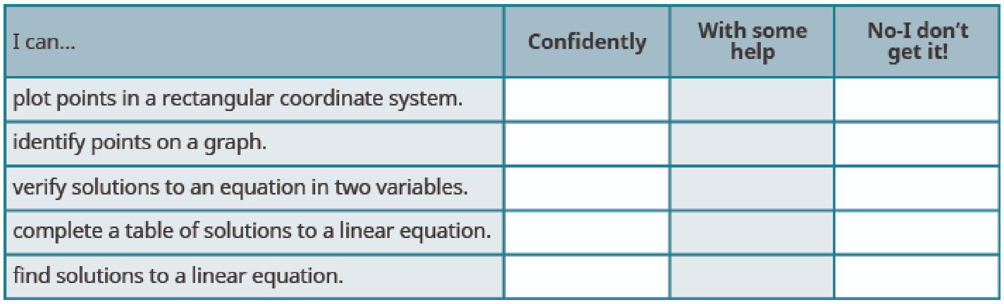
Answers will vary.

Exercise:

Problem: Is the point $(0,8)$ on the *x*-axis or *y*-axis? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no, I don't get it. This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

linear equation	A linear equation is of the form $Ax+By=C$, where A and B are not both zero, is called a linear equation in two variables.
ordered pair	An ordered pair (x,y) gives the coordinates of a point in a rectangular coordinate system.
origin	The point $(0,0)$ is called the origin. It is the point where the *x*-axis and *y*-axis intersect.
quadrant	The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.
rectangular coordinate system	

A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane’.

x-coordinate

The first number in an ordered pair (x, y).

y-coordinate

The second number in an ordered pair (x, y).

Using Interval Notation

Interval notation is another method for writing domain and range.

In set builder notation braces (curly parentheses {}) and variables are used to express the domain and range. Interval notation is often considered more efficient.

In interval notation, there are only 5 symbols to know:

- Open parentheses ( )
- Closed parentheses [ ]
- Infinity ∞
- Negative Infinity $-\infty$
- Union Sign $\cup$

To use interval notation:

Use the open parentheses ( ) if the value is not included in the graph. (i.e. the graph is undefined at that point... there's a hole or asymptote, or a jump)

If the graph goes on forever to the left, the domain will start with ($-\infty$. If the graph travels downward forever, the range will start with ($-\infty$. Similarly, if the graph goes on forever at the right or up, end with ∞)

Use the brackets [ ] if the value is part of the graph.

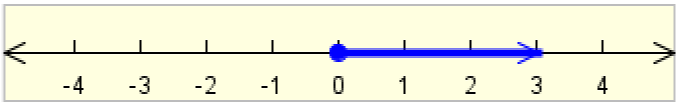
Whenever there is a break in the graph, write the interval up to the point. Then write another interval for the section of the graph after that part. Put a union sign between each interval to "join" them together.

Now for some practice so you can see if any of this makes sense.

Write the following using interval notation:

Exercise:

Problem:

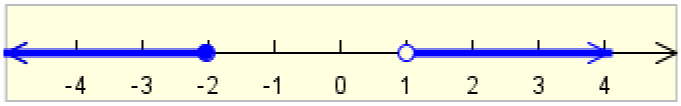


Solution:

$[0, \infty)$

Exercise:

Problem:

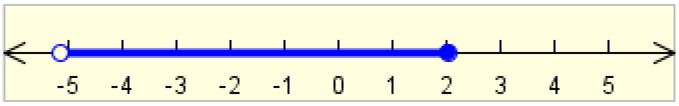


Solution:

$(-\infty, -2] \cup (1, \infty)$

Exercise:

Problem:

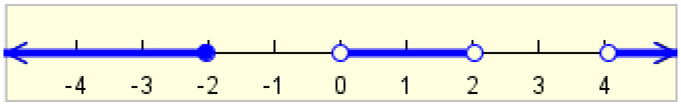


Solution:

$(-5, 2]$

Exercise:

Problem:

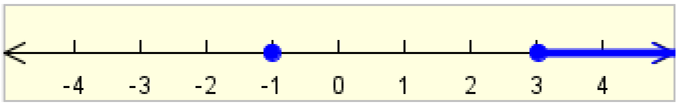


Solution:

$(-\infty, -2] \cup (0, 2) \cup (4, \infty)$

Exercise:

Problem:

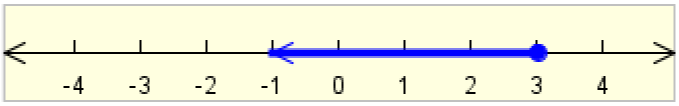


Solution:

$[-1] \cup [3, \infty)$

Exercise:

Problem:



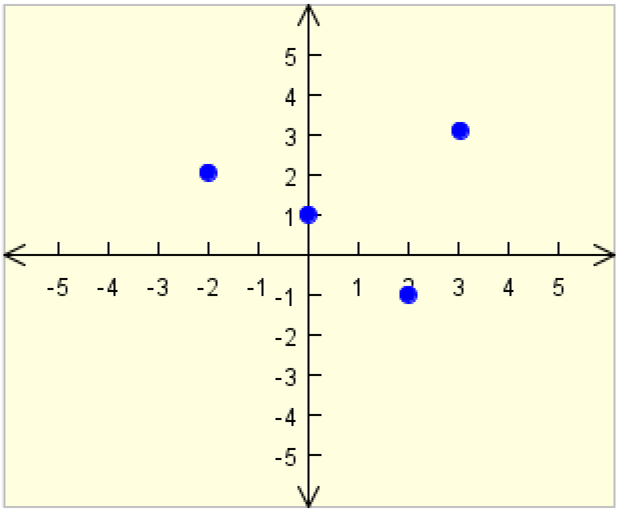
Solution:

$(-\infty, 3]$

Write the domain and range of the following in interval notation:

Exercise:

Problem:



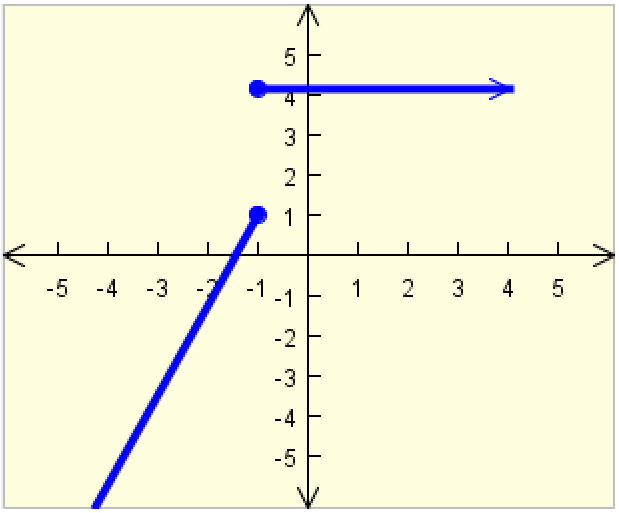
Solution:

Domain: $[-2] \cup [0] \cup [2] \cup [3]$

Range: $[-1] \cup [1] \cup [2] \cup [3]$

Exercise:

Problem:



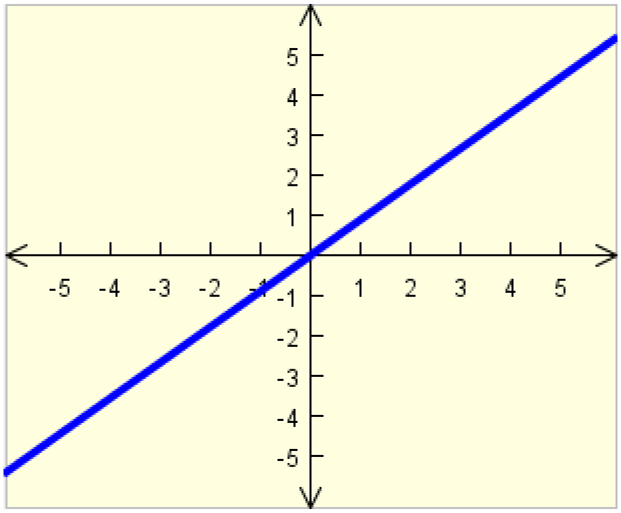
Solution:

Domain: $(-\infty, \infty)$

Range: $(\infty, 1] \cup [4]$

Exercise:

Problem:



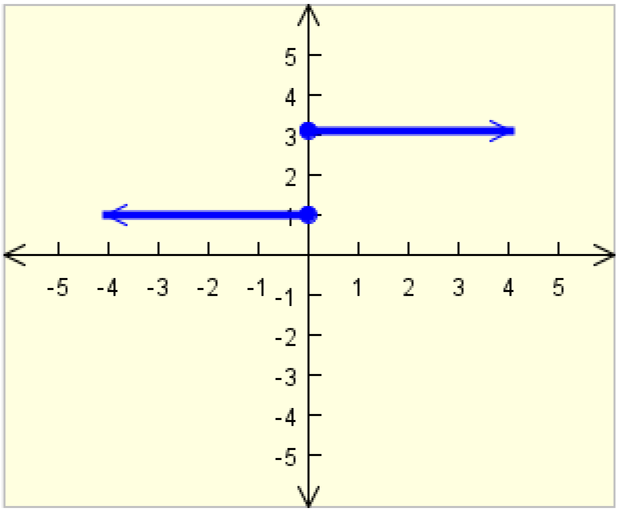
Solution:

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Exercise:

Problem:



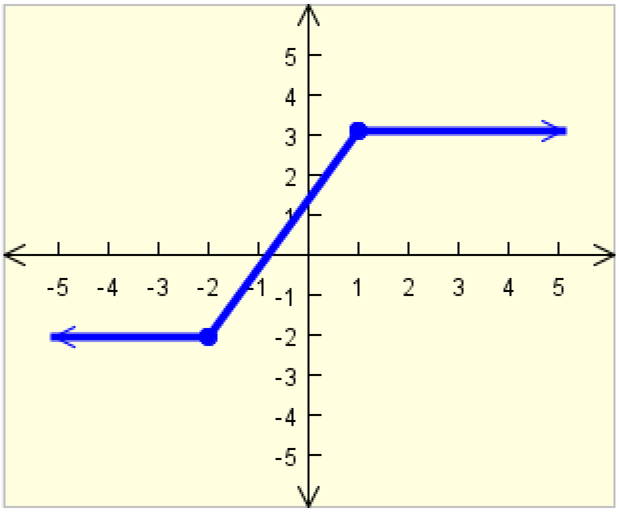
Solution:

Domain: $(-\infty, \infty)$

Range: $[1] \cup [3]$

Exercise:

Problem:



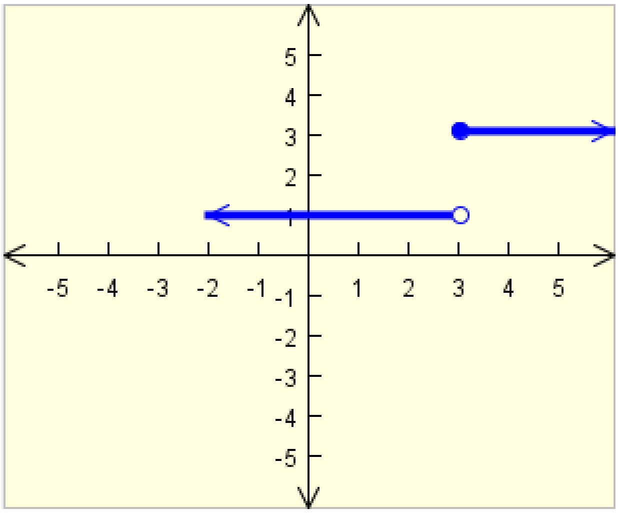
Solution:

Domain: $(-\infty, \infty)$

Range: $[-2, 3]$

Exercise:

Problem:



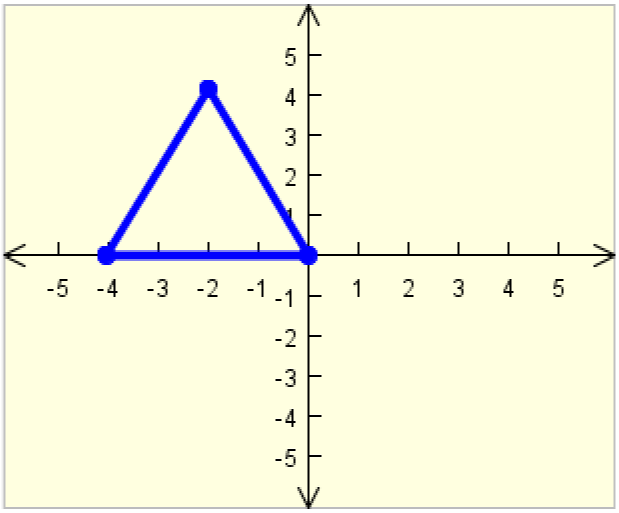
Solution:

Domain: $(-\infty, \infty)$

Range: $[1] \cup [3]$

Exercise:

Problem:



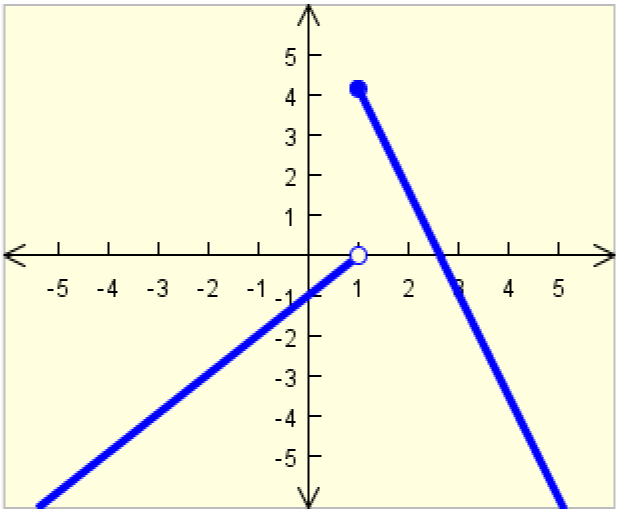
Solution:

Domain: $[-4, 0]$

Range: $[0, 4]$

Exercise:

Problem:



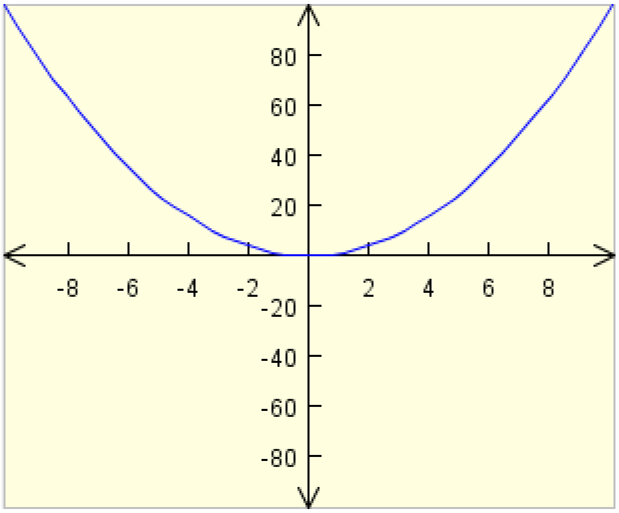
Solution:

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

Exercise:

Problem:



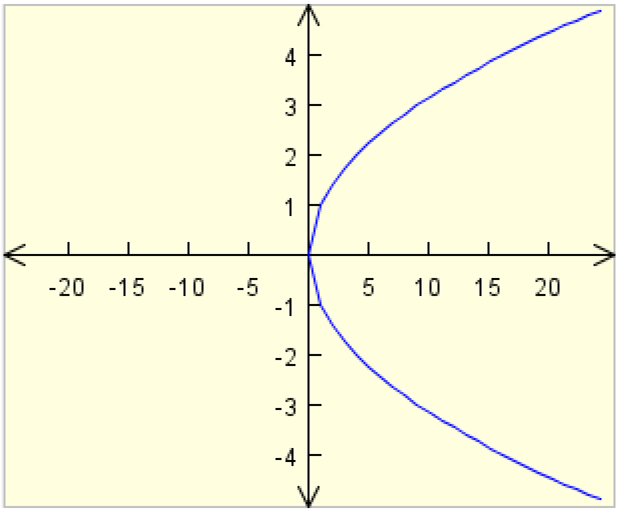
Solution:

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Exercise:

Problem:



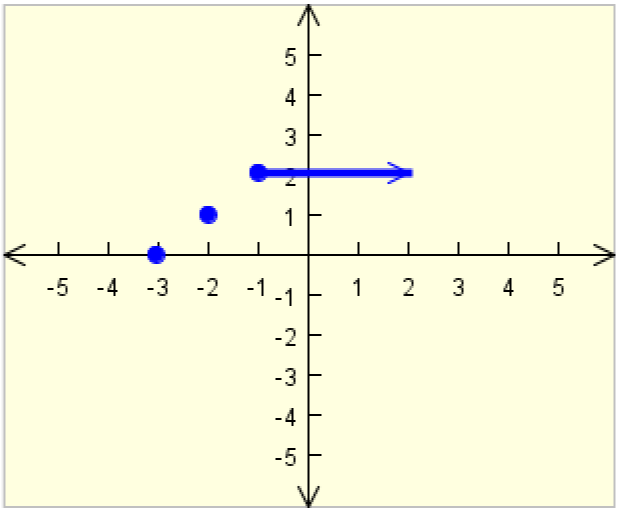
Solution:

Domain: $[0, \infty)$

Range: $(-\infty, \infty)$

Exercise:

Problem:



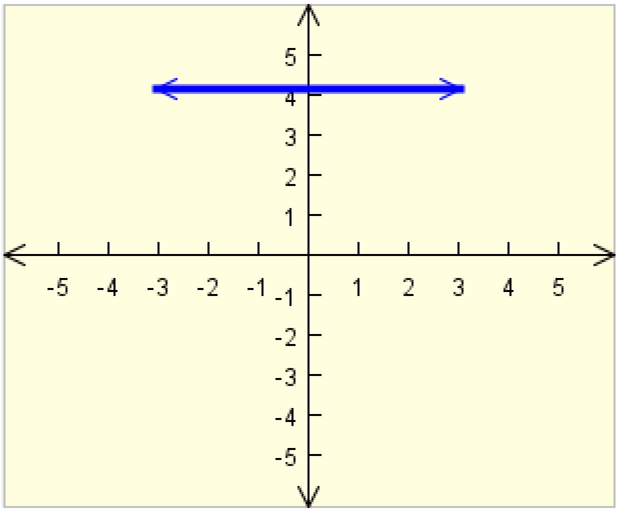
Solution:

Domain: $[-3] \cup [-2] \cup [-1, \infty)$

Range: $[0] \cup [1] \cup [2]$

Exercise:

Problem:



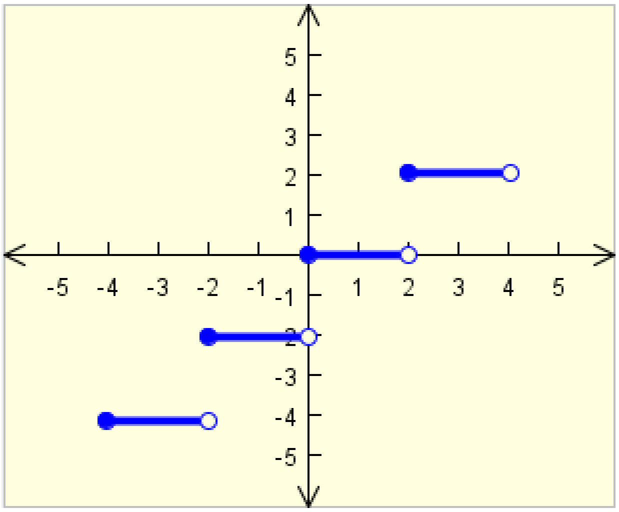
Solution:

Domain: $(-\infty, \infty)$

Range: $[4]$

Exercise:

Problem:



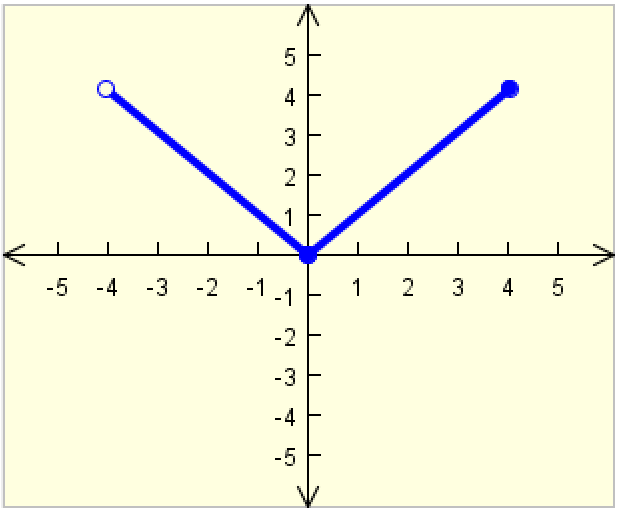
Solution:

Domain: $[-4, 4)$

Range: $[-4] \cup [-2] \cup [0] \cup [2]$

Exercise:

Problem:



Solution:

Domain: $(-4, 4]$

Range: $[0, 4]$

Interval Notation - Web Resources
Interval Notation

Interval Notation – Web Resources

Intervals @mathisfun.com

Intervals and Interval Notation @KhanAcademy

Using Interval Notation @LumenLearning

Additional Resource

https://www.youtube.com/embed/pVOHVj2aFsg

Understand Interval Notation

Solve Linear Inequalities

By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve inequalities using the Subtraction and Addition Properties of inequality
- Solve inequalities using the Division and Multiplication Properties of inequality
- Solve inequalities that require simplification
- Translate to an inequality and solve

Note:

Before you get started, take this readiness quiz.

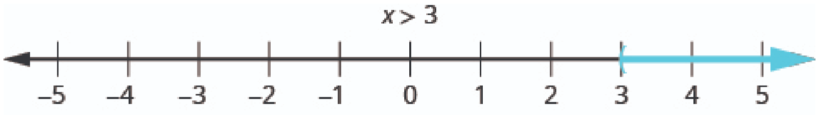
1. Translate from algebra to English: $15 > x$.
   If you missed this problem, review [link].
2. Solve: $n - 9 = -42$.
   If you missed this problem, review [link].
3. Solve: $-5p = -23$.
   If you missed this problem, review [link].
4. Solve: $3a - 12 = 7a - 20$.
   If you missed this problem, review [link].

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

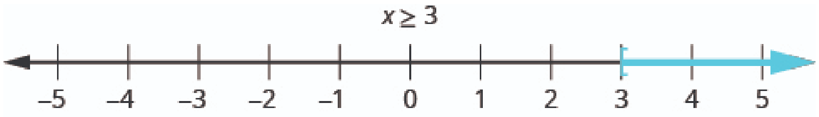
What about the solution of an inequality? What number would make the inequality $x > 3$ true? Are you thinking, '*x* could be 4'? That's correct, but *x* could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality $x > 3$.

We show the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of 3, to show that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown in [link]. Please note that the following convention is used: light blue arrows point in the positive direction and dark blue arrows point in the negative direction.



The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We do that by putting a bracket at $x = 3$, as shown in [link].



The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

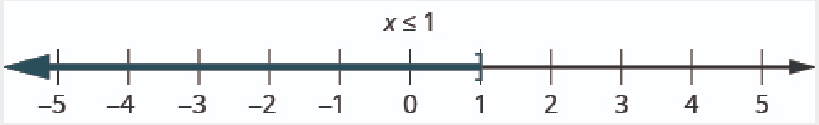
Example:
Exercise:

Problem: Graph on the number line:

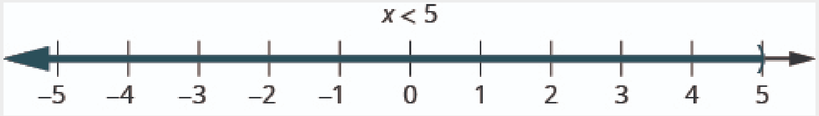
ⓐ $x \leq 1$ ⓑ $x < 5$ ⓒ $x > -1$

Solution:
Solution

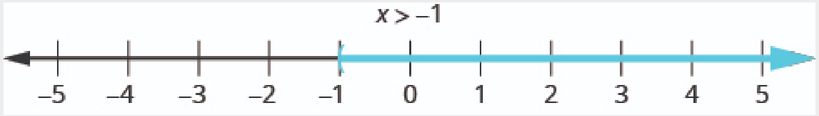
ⓐ $x \leq 1$
This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at $x = 1$ to show that it is included.



ⓑ $x < 5$
This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put a parenthesis at $x = 5$ to show it is not included.



ⓒ $x > -1$
This means all numbers greater than -1, but not including -1. We shade in all the numbers on the number line to the right of -1, then put a parenthesis at $x = -1$ to show it is not included.

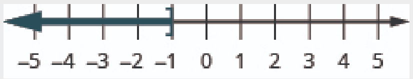


Note:
Exercise:

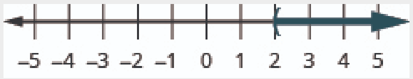
Problem: Graph on the number line: ⓐ $x \leq -1$ ⓑ $x > 2$ ⓒ $x < 3$

Solution:

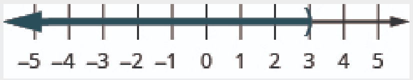
ⓐ



ⓑ



ⓒ

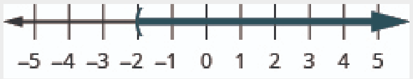


Note:
Exercise:

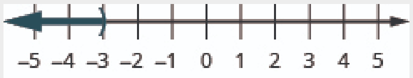
Problem: Graph on the number line: ⓐ $x > -2$ ⓑ $x < -3$ ⓒ $x \geq -1$

Solution:

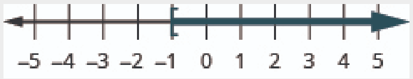
ⓐ



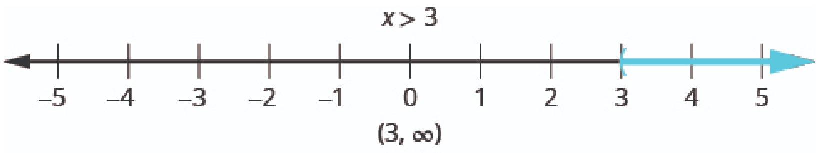
ⓑ



ⓒ

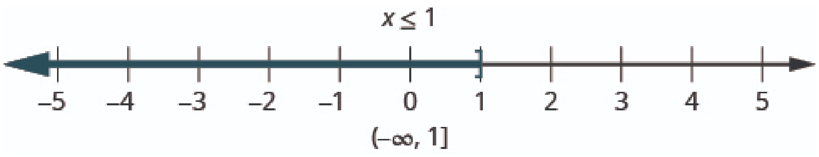


We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In interval notation, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as 'infinity'. It is not an actual number. [link] shows both the number line and the interval notation.



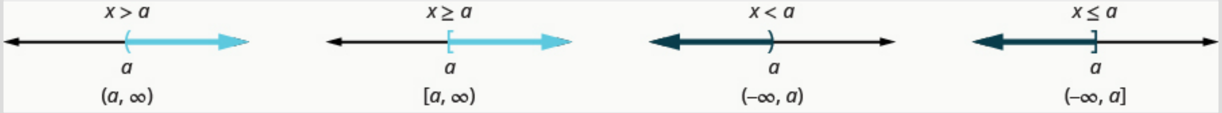
The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as ‘negative infinity’. [link] shows both the number line and interval notation.

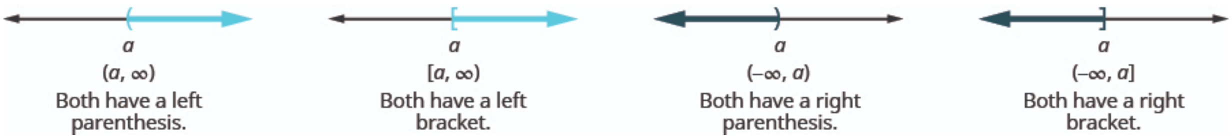


The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

Note:
Inequalities, Number Lines, and Interval Notation



Did you notice how the parenthesis or bracket in the interval notation matches the symbol at the endpoint of the arrow? These relationships are shown in [link].


a
(a, ∞)
Both have a left parenthesis.
a
$[a, \infty)$
Both have a left bracket.
a
$(-\infty, a)$
Both have a right parenthesis.
a
$(-\infty, a]$
Both have a right bracket.

The notation for inequalities on a number line and in interval notation use similar symbols to express the endpoints of intervals.

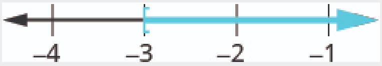
Example:
Exercise:

Problem: Graph on the number line and write in interval notation.

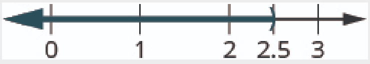
ⓐ $x \geq -3$ ⓑ $x < 2.5$ ⓒ $x \leq -\frac{3}{5}$

Solution:
Solution

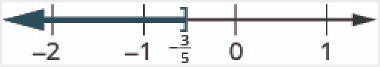
ⓐ

	$x \geq -3$
Shade to the right of -3, and put a bracket at -3.	
Write in interval notation.	$[-3, \infty)$

ⓑ

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	
Write in interval notation.	$(-\infty, 2.5)$

ⓒ

	$x \le -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

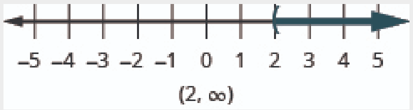
Note:
Exercise:

Problem: Graph on the number line and write in interval notation:

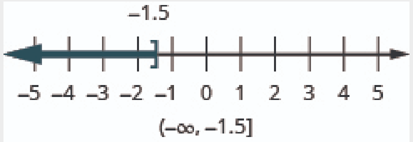
ⓐ $x > 2$ ⓑ $x \le -1.5$ ⓒ $x \ge \frac{3}{4}$

Solution:

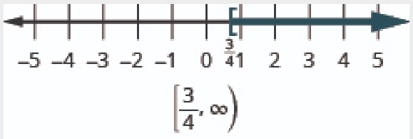
ⓐ



ⓑ



ⓒ


–5 –4 –3 –2 –1 0 $\frac{3}{4}$ 1 2 3 4 5
$\left[\frac{3}{4}, \infty\right)$

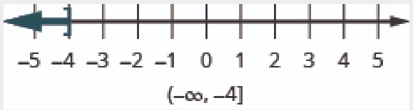
Note:
Exercise:

Problem: Graph on the number line and write in interval notation:

ⓐ $x \le -4$ ⓑ $x \ge 0.5$ ⓒ $x < -\frac{2}{3}$

Solution:

ⓐ



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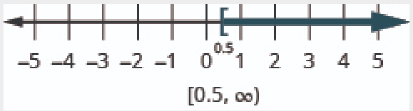

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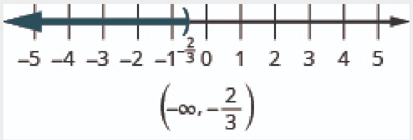

–5 –4 –3 –2 –1 $-\frac{2}{3}$ 0 1 2 3 4 5
$\left(-\infty, -\frac{2}{3}\right)$

Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal.

Note:
Properties of Equality
Equation:

Subtraction Property of Equality

For any numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a-c & = & b-c. \end{array}$$

Addition Property of Equality

For any numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a+c & = & b+c. \end{array}$$

Similar properties hold true for inequalities.

For example, we know that −4 is less than 2.	$-4 < 2$
If we subtract 5 from both quantities, is the left side still less than the right side?	$-4-5 \; ? \; 2-5$
We get −9 on the left and −3 on the right.	$-9 \; ? \; -3$
And we know −9 is less than −3.	$-9 < -3$
	The inequality sign stayed the same.

Similarly we could show that the inequality also stays the same for addition.

This leads us to the Subtraction and Addition Properties of Inequality.

Note:
Properties of Inequality
Equation:

Subtraction Property of Inequality

For any numbers $a, b,$ and c,

$$\begin{array}{lrcl} \text{if} & a & < & b \\ \text{then} & a - c & < & b - c. \end{array}$$

$$\begin{array}{lrcl} \text{if} & a & > & b \\ \text{then} & a - c & > & b - c. \end{array}$$

Addition Property of Inequality

For any numbers $a, b,$ and c,

$$\begin{array}{lrcl} \text{if} & a & < & b \\ \text{then} & a + c & < & b + c. \end{array}$$

$$\begin{array}{lrcl} \text{if} & a & > & b \\ \text{then} & a + c & > & b + c. \end{array}$$

We use these properties to solve inequalities, taking the same steps we used to solve equations. Solving the inequality $x + 5 > 9$, the steps would look like this:

Equation:

$$\begin{array}{lrcl} & x + 5 & > & 9 \\ \text{Subtract 5 from both sides to isolate } x. & x + 5 - 5 & > & 9 - 5 \\ \text{Simplify.} & x & > & 4 \end{array}$$

Any number greater than 4 is a solution to this inequality.

Example:

Exercise:

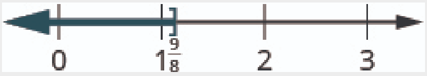
Problem:

Solve the inequality $n - \frac{1}{2} \leq \frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution:

Solution

	$n - \frac{1}{2} \leq \frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	

	$n-\frac{1}{2}+\frac{1}{2}\le\frac{5}{8}+\frac{1}{2}$
Simplify.	$n\le\frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left(-\infty,\frac{9}{8}\right]$

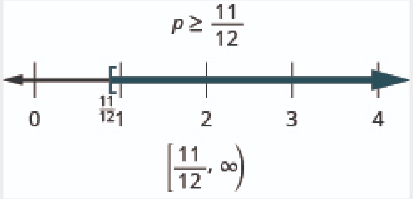
Note:
Exercise:

Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$p-\frac{3}{4}\ge\frac{1}{6}$

Solution:



$p\ge\frac{11}{12}$

$\left[\frac{11}{12},\infty\right)$

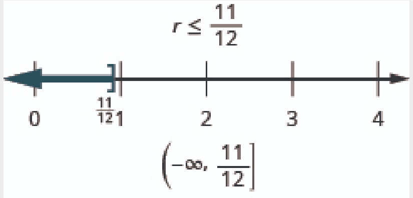
Note:
Exercise:

Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$r - \frac{1}{3} \leq \frac{7}{12}$

Solution:



Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don’t divide by 0).

Note:
Properties of Equality
Equation:

Division Property of Equality

For any numbers $a, b, c,$ and $c \neq 0,$

$$\begin{array}{llll} \text{if} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c}. \end{array}$$

Multiplication Property of Equality

For any real numbers $a, b, c,$

$$\begin{array}{llll} \text{if} & a & = & b, \\ \text{then} & ac & = & bc. \end{array}$$

Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples.

	$10 < 15$		$10 < 15$
Divide both sides by 5.	$\frac{10}{5} ? \frac{15}{5}$	Multiply both sides by 5.	$10(5) ? 15(5)$
Simplify.	$2 ? 3$		$50 ? 75$
Fill in the inequality signs.	$2 < 3$		$50 < 75$

Equation:

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

	$10 < 15$		$10 < 15$
Divide both sides by −5.	$\frac{10}{-5} ? \frac{15}{-5}$	Multiply both sides by −5.	$10(-5) ? 15(-5)$
Simplify.	$-2 ? -3$		$-50 ? -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

Equation:

The inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

Note:
Division and Multiplication Properties of Inequality
Equation:

$$\text{For any real numbers } a, b, c$$

$$\begin{array}{ll} \text{if } a < b \text{ and } c > 0, \text{then} & \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc. \\ \text{if } a > b \text{ and } c > 0, \text{then} & \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc. \\ \text{if } a < b \text{ and } c < 0, \text{then} & \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc. \\ \text{if } a > b \text{ and } c < 0, \text{then} & \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc. \end{array}$$

When we **divide or multiply** an inequality by a:

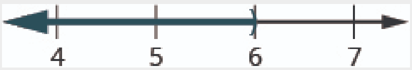
- **positive** number, the inequality stays the **same**.
- **negative** number, the inequality **reverses**.

Example:
Exercise:

Problem:

Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

Note:
Exercise:

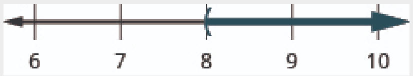
Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$9c > 72$

Solution:

$c > 8$



Note:
Exercise:

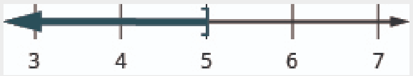
Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$12d \leq 60$

Solution:

$d \leq 5$



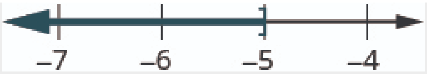
Example:
Exercise:

Problem:

Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$-10a \geq 50$
Divide both sides of the inequality by -10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	

	
Write the solution in interval notation.	$(-\infty, -5]$

Note:
Exercise:

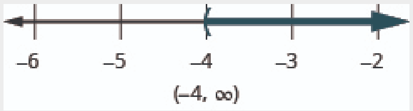
Problem:

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-8q < 32$

Solution:

$q > -4$



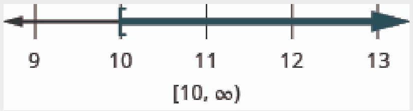
Note:
Exercise:

Problem:

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-7r \leq -70$

Solution:



Note:
Solving Inequalities
Sometimes when solving an inequality, the variable ends up on the right. We can rewrite the inequality in reverse to get the variable to the left.
Equation:

$$x > a \text{ has the same meaning as } a < x$$

Think about it as “If Xavier is taller than Alex, then Alex is shorter than Xavier.”

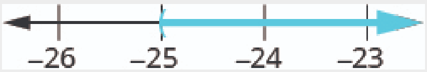
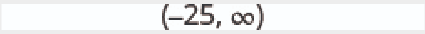
Example:
Exercise:

Problem:

Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$

Graph the solution on the number line.	
Write the solution in interval notation.	 (–25, ∞)

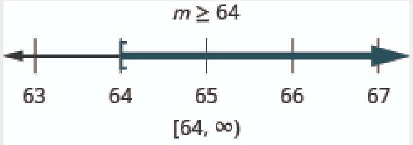
Note:
Exercise:

Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$24 \leq \frac{3}{8}m$

Solution:



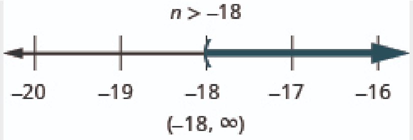
Note:
Exercise:

Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$-24 < \frac{4}{3}n$

Solution:

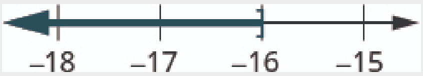


Example:
Exercise:

Problem:

Solve the inequality $\frac{t}{-2} \geq 8$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$\frac{t}{-2} \geq 8$
Multiply both sides of the inequality by -2. Since $-2 < 0$, the inequality reverses.	$-2\left(\frac{t}{-2}\right) \leq -2(8)$
Simplify.	$t \leq -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

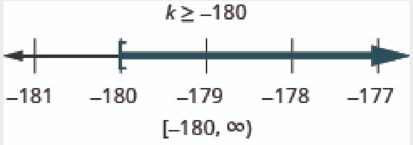
Note:
Exercise:

Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{k}{-12} \leq 15$

Solution:



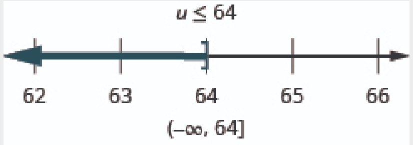
Note:
Exercise:

Problem:

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{u}{-4} \geq -16$

Solution:



Solve Inequalities That Require Simplification

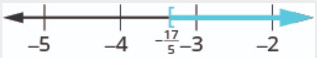
Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

Example:
Exercise:

Problem:

Solve the inequality $4m \le 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

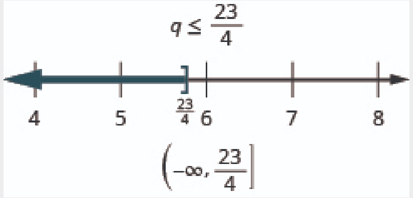
	$4m \le 9m + 17$
Subtract $9m$ from both sides to collect the variables on the left.	$4m - 9m \le 9m - 9m + 17$
Simplify.	$-5m \le 17$
Divide both sides of the inequality by -5, and reverse the inequality.	$\frac{-5m}{-5} \ge \frac{17}{-5}$
Simplify.	$m \ge -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Note:
Exercise:

Problem:

Solve the inequality $3q \geq 7q - 23$, graph the solution on the number line, and write the solution in interval notation.

Solution:

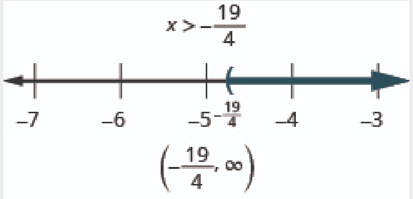


Note:
Exercise:

Problem:

Solve the inequality $6x < 10x + 19$, graph the solution on the number line, and write the solution in interval notation.

Solution:

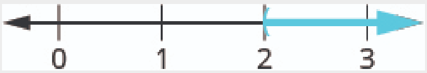


Example:
Exercise:

Problem:

Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

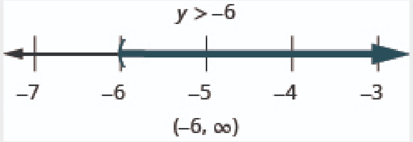
Simplify each side as much as possible.	$8p + 3(p - 12) > 7p - 28$
Distribute.	$8p + 3p - 36 > 7p - 28$
Combine like terms.	$11p - 36 > 7p - 28$
Subtract $7p$ from both sides to collect the variables on the left.	$11p - 36 - 7p > 7p - 28 - 7p$
Simplify.	$4p - 36 > -28$
Add 36 to both sides to collect the constants on the right.	$4p - 36 + 36 > -28 + 36$
Simplify.	$4p > 8$
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > \frac{8}{4}$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interal notation.	$(2, \infty)$

Note:
Exercise:

Problem:

Solve the inequality $9y + 2(y + 6) > 5y - 24$, graph the solution on the number line, and write the solution in interval notation.

Solution:

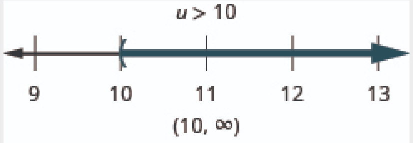


Note:
Exercise:

Problem:

Solve the inequality $6u + 8(u - 1) > 10u + 32$, graph the solution on the number line, and write the solution in interval notation.

Solution:



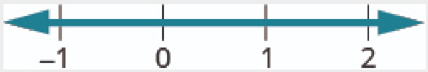
Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

Example:
Exercise:

Problem:

Solve the inequality $8x - 2\left(5 - x\right) < 4\left(x + 9\right) + 6x$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

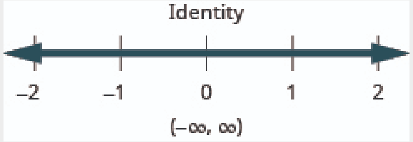
Simplify each side as much as possible.	$8x - 2(5 - x) < 4(x + 9) + 6x$
Distribute.	$8x - 10 + 2x < 4x + 36 + 6x$
Combine like terms.	$10x - 10 < 10x + 36$
Subtract $10x$ from both sides to collect the variables on the left.	$10x - 10 - 10x < 10x + 36 - 10x$
Simplify.	$-10 < 36$
The x’s are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \infty)$

Note:
Exercise:

Problem:

Solve the inequality $4b - 3(3 - b) > 5(b - 6) + 2b$, graph the solution on the number line, and write the solution in interval notation.

Solution:

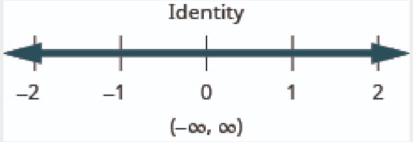


Note:
Exercise:

Problem:

Solve the inequality $9h - 7\left(2 - h\right) < 8\left(h + 11\right) + 8h$, graph the solution on the number line, and write the solution in interval notation.

Solution:

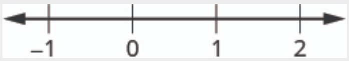


Example:
Exercise:

Problem:

Solve the inequality $\frac{1}{3}a - \frac{1}{8}a > \frac{5}{24}a + \frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

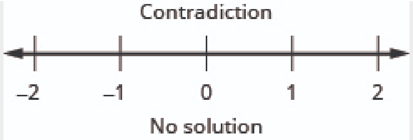
	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract $5a$ from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Note:
Exercise:

Problem:

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution:

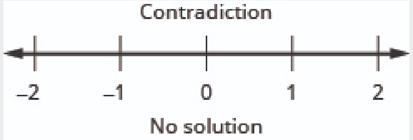


Note:
Exercise:

Problem:

Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Solution:



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like ‘more than’ and ‘less than’. But others are not as obvious.

Think about the phrase ‘at least’ – what does it mean to be ‘at least 21 years old’? It means 21 or more. The phrase ‘at least’ is the same as ‘greater than or equal to’.

[link] shows some common phrases that indicate inequalities.

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to

$>$	$\geq$	$<$	$\leq$
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

Example:
Exercise:

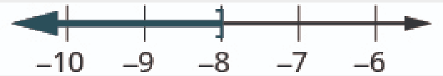
Problem:

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twelve times *c* is no more than 96.

Solution:
Solution

Translate.	$\text{Twelve times } c \underbrace{\text{ is no more than }} 96$ $12c \leq 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \leq \frac{96}{12}$
Simplify.	$c \leq 8$
Write in interval notation.	$(-\infty, -8]$

Graph on the number line.	

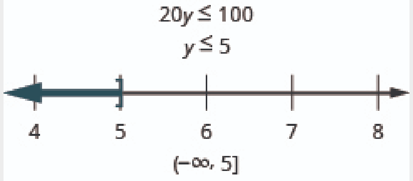
Note:
Exercise:

Problem:

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twenty times y is at most 100

Solution:



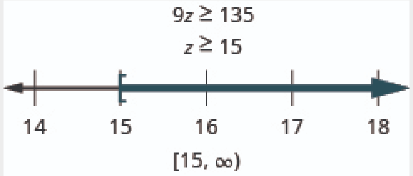
Note:
Exercise:

Problem:

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nine times z is no less than 135

Solution:



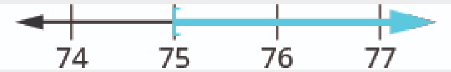
Example:
Exercise:

Problem:

Translate and solve. Then write the solution in interval notation and graph on the number line.

Thirty less than *x* is at least 45.

Solution:
Solution

Translate.	Thirty less than x is at least 45. $x - 30 \geq 45$
Solve—add 30 to both sides.	$x - 30 + 30 \geq 45 + 30$
Simplify.	$x \geq 75$
Write in interval notation.	$[75, \infty)$
Graph on the number line.	

Note:

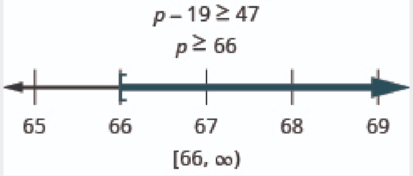
Exercise:

Problem:

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nineteen less than *p* is no less than 47

Solution:



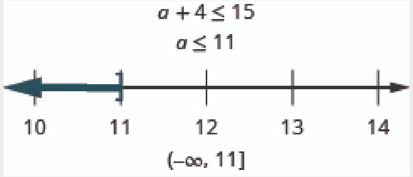
Note:
Exercise:

Problem:

Translate and solve. Then write the solution in interval notation and graph on the number line.

Four more than *a* is at most 15.

Solution:



Key Concepts

- **Subtraction Property of Inequality**
  For any numbers a, b, and c,
  if $a < b$ then $a - c < b - c$ and
  if $a > b$ then $a - c > b - c$.

- **Addition Property of Inequality**
  For any numbers a, b, and c,
  if $a < b$ then $a + c < b + c$ and
  if $a > b$ then $a + c > b + c$.
- **Division and Multiplication Properties of Inequality**
  For any numbers a, b, and c,
  if $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.
  if $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.
  if $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.
  if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.
- When we **divide or multiply** an inequality by a:
  - **positive** number, the inequality stays the **same**.
  - **negative** number, the inequality **reverses**.

Practice Makes Perfect

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

Exercise:

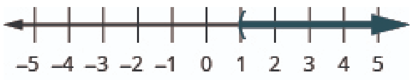
ⓐ $x \leq 2$
ⓑ $x > -1$
Problem: ⓒ $x < 0$

Exercise:

ⓐ $x > 1$
ⓑ $x < -2$
Problem: ⓒ $x \geq -3$

Solution:

ⓐ



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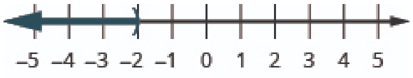

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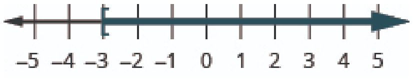


Exercise:

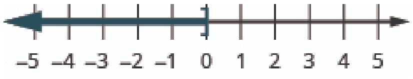
ⓐ $x \geq -3$
ⓑ $x < 4$
Problem: ⓒ $x \leq -2$

Exercise:

ⓐ $x \leq 0$
ⓑ $x > -4$
Problem: ⓒ $x \geq -1$

Solution:

ⓐ



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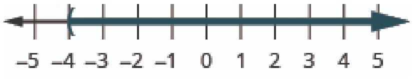

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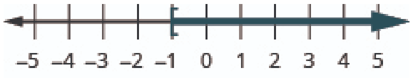


In the following exercises, graph each inequality on the number line and write in interval notation.

Exercise:

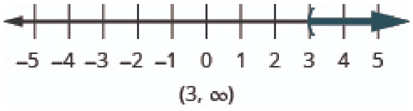
Problem: ⓐ $x < -2$
ⓑ $x \geq -3.5$
ⓒ $x \leq \frac{2}{3}$

Exercise:

Problem: ⓐ $x > 3$
ⓑ $x \leq -0.5$
ⓒ $x \geq \frac{1}{3}$

Solution:

ⓐ



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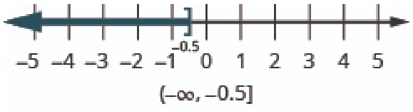

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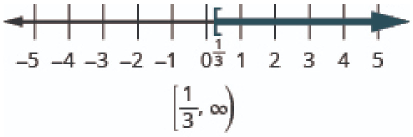


Exercise:

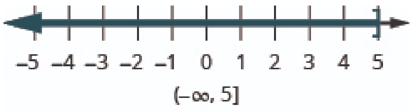
Problem: ⓐ $x \geq -4$
ⓑ $x < 2.5$
ⓒ $x > -\frac{3}{2}$

Exercise:

ⓐ $x \le 5$
ⓑ $x \ge -1.5$
Problem: ⓒ $x < -\frac{7}{3}$

Solution:

ⓐ



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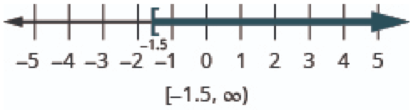

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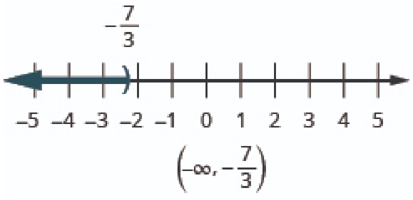


Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

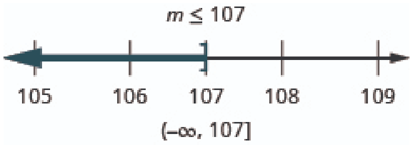
Exercise:

Problem: $n - 11 < 33$

Exercise:

Problem: $m - 45 \le 62$

Solution:



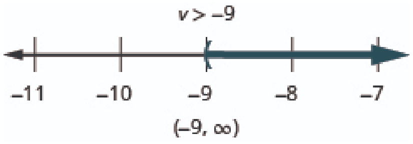
Exercise:

Problem: $u + 25 > 21$

Exercise:

Problem: $v + 12 > 3$

Solution:



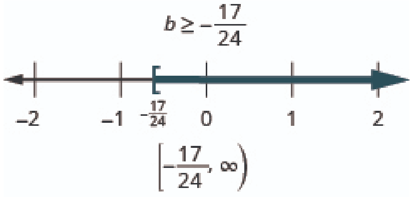
Exercise:

Problem: $a + \frac{3}{4} \geq \frac{7}{10}$

Exercise:

Problem: $b + \frac{7}{8} \geq \frac{1}{6}$

Solution:



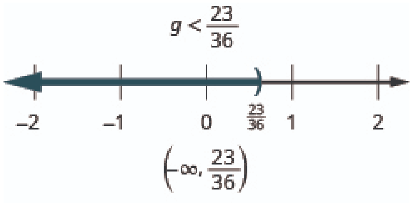
Exercise:

Problem: $f - \frac{13}{20} < -\frac{5}{12}$

Exercise:

Problem: $g - \frac{11}{12} < -\frac{5}{18}$

Solution:



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

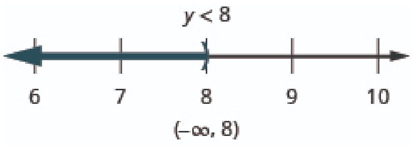
Exercise:

Problem: $8x > 72$

Exercise:

Problem: $6y < 48$

Solution:



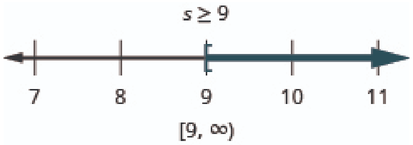
Exercise:

Problem: $7r \leq 56$

Exercise:

Problem: $9s \geq 81$

Solution:



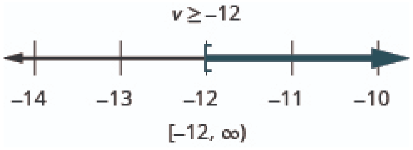
Exercise:

Problem: $-5u \geq 65$

Exercise:

Problem: $-8v \leq 96$

Solution:



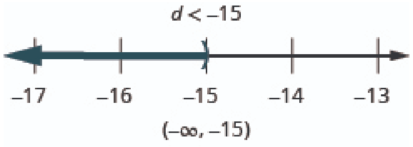
Exercise:

Problem: $-9c < 126$

Exercise:

Problem: $-7d > 105$

Solution:



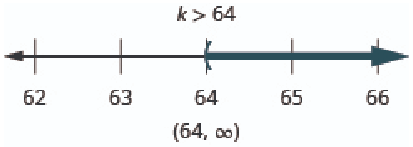
Exercise:

Problem: $20 > \frac{2}{5}h$

Exercise:

Problem: $40 < \frac{5}{8}k$

Solution:



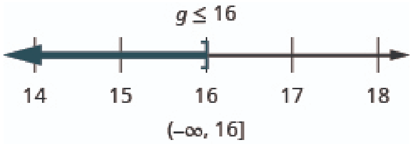
Exercise:

Problem: $\frac{7}{6}j \geq 42$

Exercise:

Problem: $\frac{9}{4}g \leq 36$

Solution:



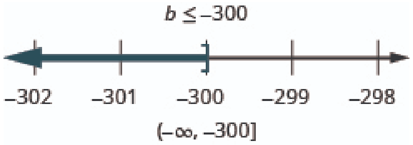
Exercise:

Problem: $\frac{a}{-3} \leq 9$

Exercise:

Problem: $\frac{b}{-10} \geq 30$

Solution:



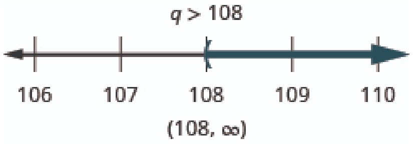
Exercise:

Problem: $-25 < \frac{p}{-5}$

Exercise:

Problem: $-18 > \frac{q}{-6}$

Solution:



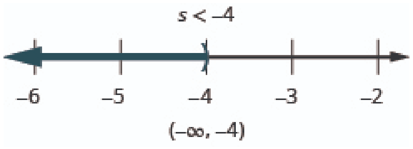
Exercise:

Problem: $9t \geq -27$

Exercise:

Problem: $7s < -28$

Solution:



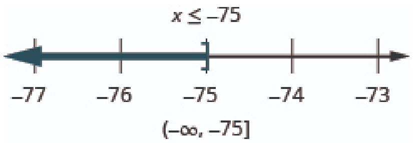
Exercise:

Problem: $\frac{2}{3}y > -36$

Exercise:

Problem: $\frac{3}{5}x \leq -45$

Solution:



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

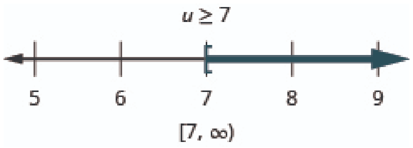
Exercise:

Problem: $4v \geq 9v - 40$

Exercise:

Problem: $5u \leq 8u - 21$

Solution:



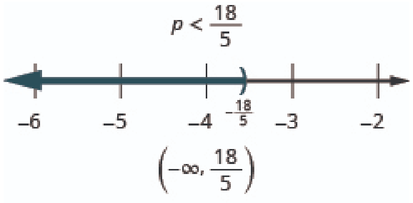
Exercise:

Problem: $13q < 7q - 29$

Exercise:

Problem: $9p > 14p - 18$

Solution:



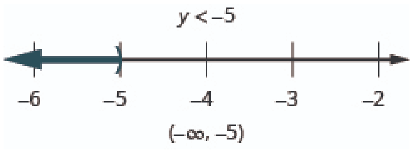
Exercise:

Problem: $12x + 3(x + 7) > 10x - 24$

Exercise:

Problem: $9y + 5(y + 3) < 4y - 35$

Solution:



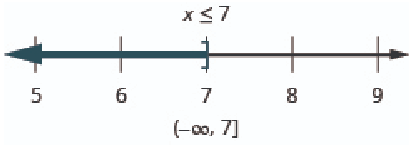
Exercise:

Problem: $6h - 4(h - 1) \leq 7h - 11$

Exercise:

Problem: $4k - (k - 2) \geq 7k - 26$

Solution:



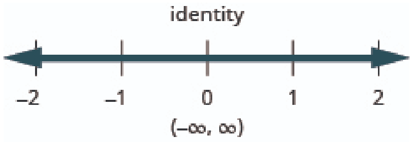
Exercise:

Problem: $8m-2\left(14-m\right)\ge 7\left(m-4\right)+3m$

Exercise:

Problem: $6n-12\left(3-n\right)\le 9\left(n-4\right)+9n$

Solution:



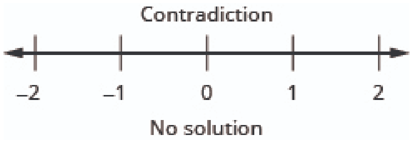
Exercise:

Problem: $\frac{3}{4}b-\frac{1}{3}b<\frac{5}{12}b-\frac{1}{2}$

Exercise:

Problem: $9u+5(2u-5)\ge 12(u-1)+7u$

Solution:



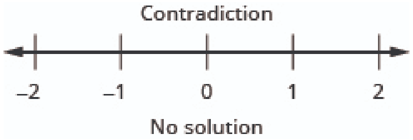
Exercise:

Problem: $\frac{2}{3}g-\frac{1}{2}(g-14)\le \frac{1}{6}(g+42)$

Exercise:

Problem: $\frac{5}{6}a-\frac{1}{4}a>\frac{7}{12}a+\frac{2}{3}$

Solution:



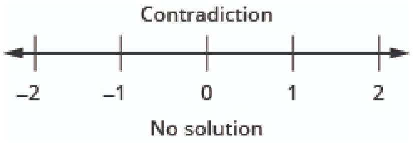
Exercise:

Problem: $\frac{4}{5}h - \frac{2}{3}(h - 9) \geq \frac{1}{15}(2h + 90)$

Exercise:

Problem: $12v + 3(4v - 1) \leq 19(v - 2) + 5v$

Solution:



Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

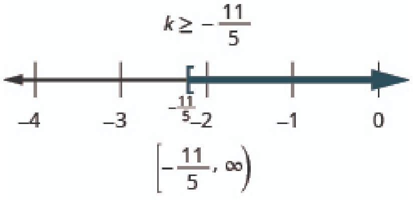
Exercise:

Problem: $15k \leq -40$

Exercise:

Problem: $35k \geq -77$

Solution:



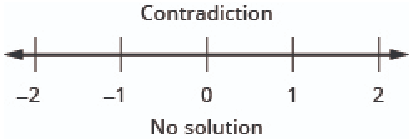
Exercise:

Problem: $23p-2(6-5p)>3(11p-4)$

Exercise:

Problem: $18q-4(10-3q)<5(6q-8)$

Solution:



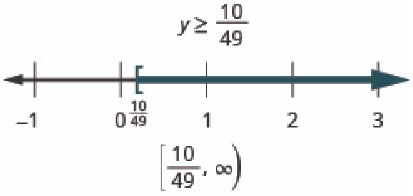
Exercise:

Problem: $-\frac{9}{4}x\ge -\frac{5}{12}$

Exercise:

Problem: $-\frac{21}{8}y\le -\frac{15}{28}$

Solution:



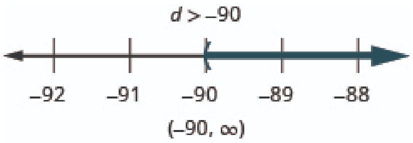
Exercise:

Problem: $c+34<-99$

Exercise:

Problem: $d+29>-61$

Solution:



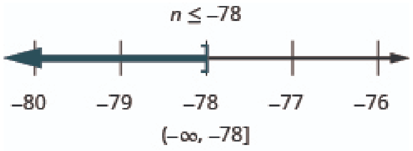
Exercise:

Problem: $\frac{m}{18} \geq -4$

Exercise:

Problem: $\frac{n}{13} \leq -6$

Solution:



Translate to an Inequality and Solve

In the following exercises, translate and solve .Then write the solution in interval notation and graph on the number line.

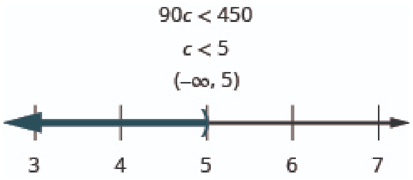
Exercise:

Problem: Fourteen times *d* is greater than 56.

Exercise:

Problem: Ninety times *c* is less than 450.

Solution:



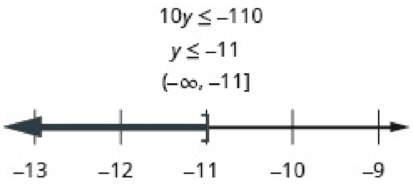
Exercise:

Problem: Eight times *z* is smaller than -40.

Exercise:

Problem: Ten times *y* is at most -110.

Solution:



$10y \leq -110$

$y \leq -11$

$(-\infty, -11]$

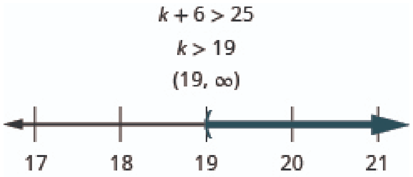
Exercise:

Problem: Three more than *h* is no less than 25.

Exercise:

Problem: Six more than *k* exceeds 25.

Solution:



$k + 6 > 25$

$k > 19$

$(19, \infty)$

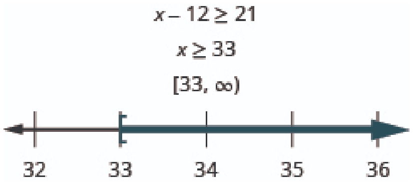
Exercise:

Problem: Ten less than *w* is at least 39.

Exercise:

Problem: Twelve less than *x* is no less than 21.

Solution:



$x - 12 \geq 21$

$x \geq 33$

$[33, \infty)$

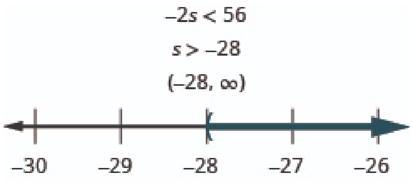
Exercise:

Problem: Negative five times *r* is no more than 95.

Exercise:

Problem: Negative two times *s* is lower than 56.

Solution:



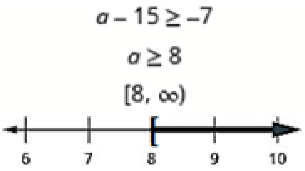
Exercise:

Problem: Nineteen less than *b* is at most -22.

Exercise:

Problem: Fifteen less than *a* is at least -7.

Solution:



Everyday Math

Exercise:

Problem:

Safety A child's height, *h*, must be at least 57 inches for the child to safely ride in the front seat of a car. Write this as an inequality.

Exercise:

Problem:

Fighter pilots The maximum height, *h*, of a fighter pilot is 77 inches. Write this as an inequality.

Solution:

$h \leq 77$

Exercise:

Problem:

Elevators The total weight, *w*, of an elevator’s passengers can be no more than 1,200 pounds. Write this as an inequality.

Exercise:

Problem:

Shopping The number of items, *n*, a shopper can have in the express check-out lane is at most 8. Write this as an inequality.

Solution:

$n \leq 8$

Writing Exercises

Exercise:

Problem: Give an example from your life using the phrase ‘at least’.

Exercise:

Problem: Give an example from your life using the phrase ‘at most’.

Solution:

Answers will vary.

Exercise:

Problem:

Explain why it is necessary to reverse the inequality when solving $-5x > 10$.

Exercise:

Problem:

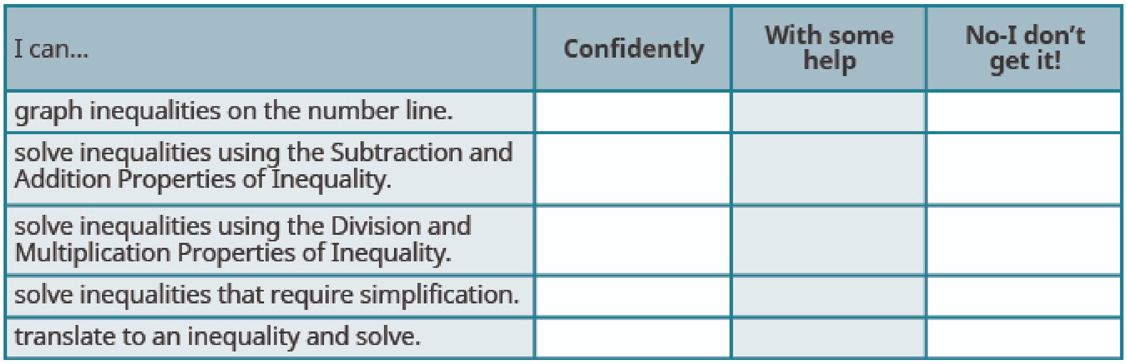
Explain why it is necessary to reverse the inequality when solving $\frac{n}{-3} < 12$.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Chapter 2 Review Exercises

Solve Equations using the Subtraction and Addition Properties of Equality

Verify a Solution of an Equation

In the following exercises, determine whether each number is a solution to the equation.

Exercise:

Problem: $10x - 1 = 5x; x = \frac{1}{5}$

Exercise:

Problem: $w + 2 = \frac{5}{8}; w = \frac{3}{8}$

Solution:

no

Exercise:

Problem: $-12n + 5 = 8n; n = -\frac{5}{4}$

Exercise:

Problem: $6a - 3 = -7a, a = \frac{3}{13}$

Solution:

yes

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction Property of Equality.

Exercise:

Problem: $x + 7 = 19$

Exercise:

Problem: $y + 2 = -6$

Solution:

$y = -8$

Exercise:

Problem: $a + \frac{1}{3} = \frac{5}{3}$

Exercise:

Problem: $n + 3.6 = 5.1$

Solution:

$n = 1.5$

In the following exercises, solve each equation using the Addition Property of Equality.

Exercise:

Problem: $u - 7 = 10$

Exercise:

Problem: $x - 9 = -4$

Solution:

$x = 5$

Exercise:

Problem: $c - \frac{3}{11} = \frac{9}{11}$

Exercise:

Problem: $p - 4.8 = 14$

Solution:

$p = 18.8$

In the following exercises, solve each equation.

Exercise:

Problem: $n - 12 = 32$

Exercise:

Problem: $y + 16 = -9$

Solution:

$y = -25$

Exercise:

Problem: $f + \frac{2}{3} = 4$

Exercise:

Problem: $d - 3.9 = 8.2$

Solution:

$d = 12.1$

Solve Equations That Require Simplification

In the following exercises, solve each equation.

Exercise:

Problem: $y + 8 - 15 = -3$

Exercise:

Problem: $7x + 10 - 6x + 3 = 5$

Solution:

$x = -8$

Exercise:

Problem: $6(n - 1) - 5n = -14$

Exercise:

Problem: $8(3p + 5) - 23(p - 1) = 35$

Solution:

$p = -28$

Translate to an Equation and Solve

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise:

Problem: The sum of -6 and m is 25.

Exercise:

Problem: Four less than n is 13.

Solution:

$n - 4 = 13; n = 17$

Translate and Solve Applications

In the following exercises, translate into an algebraic equation and solve.

Exercise:

Problem:

Rochelle’s daughter is 11 years old. Her son is 3 years younger. How old is her son?

Exercise:

Problem:

Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?

Solution:

161 pounds

Exercise:

Problem:

Peter paid $9.75 to go to the movies, which was $46.25 less than he paid to go to a concert. How much did he pay for the concert?

Exercise:

Problem:

Elissa earned $152.84 this week, which was $21.65 more than she earned last week. How much did she earn last week?

Solution:

$131.19

Solve Equations using the Division and Multiplication Properties of Equality

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the division and multiplication properties of equality and check the solution.

Exercise:

Problem: $8x = 72$

Exercise:

Problem: $13a = -65$

Solution:

$a = -5$

Exercise:

Problem: $0.25p = 5.25$

Exercise:

Problem: $-y = 4$

Solution:

$y = -4$

Exercise:

Problem: $\frac{n}{6} = 18$

Exercise:

Problem: $\frac{y}{-10} = 30$

Solution:

$y = -300$

Exercise:

Problem: $36 = \frac{3}{4}x$

Exercise:

Problem: $\frac{5}{8}u = \frac{15}{16}$

Solution:

$u = \frac{3}{2}$

Exercise:

Problem: $-18m = -72$

Exercise:

Problem: $\frac{c}{9} = 36$

Solution:

$c = 324$

Exercise:

Problem: $0.45x = 6.75$

Exercise:

Problem: $\frac{11}{12} = \frac{2}{3}y$

Solution:

$y = \frac{11}{8}$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

Exercise:

Problem: $5r - 3r + 9r = 35 - 2$

Exercise:

Problem: $24x + 8x - 11x = -7 - 14$

Solution:

$x = -1$

Exercise:

Problem: $\frac{11}{12}n - \frac{5}{6}n = 9 - 5$

Exercise:

Problem: $-9(d - 2) - 15 = -24$

Solution:

$d = 3$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise:

Problem: 143 is the product of -11 and y.

Exercise:

Problem: The quotient of b and 9 is -27.

Solution:

$\frac{b}{9} = -27; b = -243$

Exercise:

Problem: The sum of q and one-fourth is one.

Exercise:

Problem: The difference of s and one-twelfth is one fourth.

Solution:

$s - \frac{1}{12} = \frac{1}{4}; s = \frac{1}{3}$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise:

Problem:

Ray paid $21 for 12 tickets at the county fair. What was the price of each ticket?

Exercise:

Problem:

Janet gets paid $24 per hour. She heard that this is $\frac{3}{4}$ of what Adam is paid. How much is Adam paid per hour?

Solution:

$32

Solve Equations with Variables and Constants on Both Sides

Solve an Equation with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

Exercise:

Problem: $8p+7=47$

Exercise:

Problem: $10w-5=65$

Solution:

$w=7$

Exercise:

Problem: $3x+19=-47$

Exercise:

Problem: $32=-4-9n$

Solution:

$n=-4$

Solve an Equation with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

Exercise:

Problem: $7y=6y-13$

Exercise:

Problem: $5a+21=2a$

Solution:

$a = -7$

Exercise:

Problem: $k = -6k - 35$

Exercise:

Problem: $4x - \frac{3}{8} = 3x$

Solution:

$x = \frac{3}{8}$

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

Exercise:

Problem: $12x - 9 = 3x + 45$

Exercise:

Problem: $5n - 20 = -7n - 80$

Solution:

$n = -5$

Exercise:

Problem: $4u + 16 = -19 - u$

Exercise:

Problem: $\frac{5}{8}c - 4 = \frac{3}{8}c + 4$

Solution:

$c = 32$

Use a General Strategy for Solving Linear Equations

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise:

Problem: $6(x+6)=24$

Exercise:

Problem: $9(2p-5)=72$

Solution:

$p=\frac{13}{2}$

Exercise:

Problem: $-(s+4)=18$

Exercise:

Problem: $8+3(n-9)=17$

Solution:

$n=12$

Exercise:

Problem: $23-3(y-7)=8$

Exercise:

Problem: $\frac{1}{3}(6m+21)=m-7$

Solution:

$m=-14$

Exercise:

Problem: $4(3.5y+0.25)=365$

Exercise:

Problem: $0.25(q-8)=0.1(q+7)$

Solution:

$q = 18$

Exercise:

Problem: $8(r - 2) = 6(r + 10)$

Exercise:

Problem: $5 + 7(2 - 5x) = 2(9x + 1) -(13x - 57)$

Solution:

$x = -1$

Exercise:

Problem: $(9n + 5) - (3n - 7) = 20 - (4n - 2)$

Exercise:

Problem: $2[-16 + 5(8k - 6)] = 8(3 - 4k) - 32$

Solution:

$k = \frac{3}{4}$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise:

Problem: $17y - 3(4 - 2y) = 11(y - 1) +12y - 1$

Exercise:

Problem: $9u + 32 = 15(u - 4) -3(2u + 21)$

Solution:

contradiction; no solution

Exercise:

Problem: $-8(7m+4)=-6(8m+9)$

Exercise:

Problem: $21(c-1)-19(c+1)=2(c-20)$

Solution:

identity; all real numbers

Solve Equations with Fractions and Decimals

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

Exercise:

Problem: $\frac{2}{5}n-\frac{1}{10}=\frac{7}{10}$

Exercise:

Problem: $\frac{1}{3}x+\frac{1}{5}x=8$

Solution:

$x=15$

Exercise:

Problem: $\frac{3}{4}a-\frac{1}{3}=\frac{1}{2}a-\frac{5}{6}$

Exercise:

Problem: $\frac{1}{2}(k-3)=\frac{1}{3}(k+16)$

Solution:

$k = 41$

Exercise:

Problem: $\frac{3x-2}{5} = \frac{3x+4}{8}$

Exercise:

Problem: $\frac{5y-1}{3} + 4 = \frac{-8y+4}{6}$

Solution:

$y = -1$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

Exercise:

Problem: $0.8x - 0.3 = 0.7x + 0.2$

Exercise:

Problem: $0.36u + 2.55 = 0.41u + 6.8$

Solution:

$u = -85$

Exercise:

Problem: $0.6p - 1.9 = 0.78p + 1.7$

Exercise:

Problem: $0.6p - 1.9 = 0.78p + 1.7$

Solution:

$d = -20$

Solve a Formula for a Specific Variable

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise:

Problem:

Natalie drove for $7\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Exercise:

Problem:

Mallory is taking the bus from St. Louis to Chicago. The distance is 300 miles and the bus travels at a steady rate of 60 miles per hour. How long will the bus ride be?

Solution:

5 hours

Exercise:

Problem:

Aaron’s friend drove him from Buffalo to Cleveland. The distance is 187 miles and the trip took 2.75 hours. How fast was Aaron’s friend driving?

Exercise:

Problem:

Link rode his bike at a steady rate of 15 miles per hour for $2\frac{1}{2}$ hours. How much distance did he travel?

Solution:

37.5 miles

Solve a Formula for a Specific Variable

In the following exercises, solve.

Exercise:

Problem:

Use the formula. $d = rt$ to solve for t

ⓐ when $d = 510$ and $r = 60$

ⓑ in general

Exercise:

Problem: Use the formula. $d = rt$ to solve for r
ⓐ when when $d = 451$ and $t = 5.5$
ⓑ in general

Solution:

ⓐ $r = 82\,\text{mph}$; ⓑ $r = \frac{D}{t}$

Exercise:

Problem: Use the formula $A = \frac{1}{2}bh$ to solve for b
ⓐ when $A = 390$ and $h = 26$
ⓑ in general

Exercise:

Problem: Use the formula $A = \frac{1}{2}bh$ to solve for h
ⓐ when $A = 153$ and $b = 18$
ⓑ in general

Solution:

ⓐ $h = 17$ ⓑ $h = \frac{2A}{b}$

Exercise:

Problem: Use the formula $I = Prt$ to solve for the principal, P for
ⓐ $I = \$2,501, r = 4.1\%$,
$t = 5\,\text{years}$
ⓑ in general

Exercise:

Problem: Solve the formula $4x + 3y = 6$ for y
ⓐ when $x = -2$
ⓑ in general

Solution:

ⓐ $y = \frac{14}{3}$ ⓑ $y = \frac{6-4x}{3}$

Exercise:

Problem: Solve $180 = a + b + c$ for c.

Exercise:

Problem: Solve the formula $V = LWH$ for H.

Solution:

$H = \frac{V}{LW}$

Solve Linear Inequalities

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

Exercise:

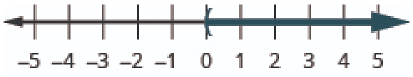
ⓐ $x \leq 4$
ⓑ $x > -2$
Problem: ⓒ $x < 1$

Exercise:

ⓐ $x > 0$
ⓑ $x < -3$
Problem: ⓒ $x \geq 1$

Solution:

ⓐ



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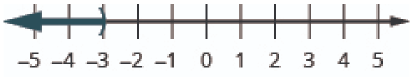

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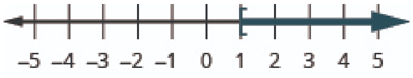


In the following exercises, graph each inequality on the number line and write in interval notation.

Exercise:

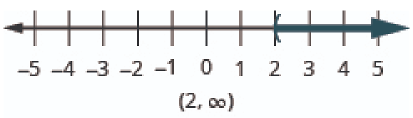
ⓐ $x < -1$
ⓑ $x \geq -2.5$
Problem: ⓒ $x \leq \frac{5}{4}$

Exercise:

ⓐ $x > 2$
ⓑ $x \leq -1.5$
Problem: ⓒ $x \geq \frac{5}{3}$

Solution:

ⓐ



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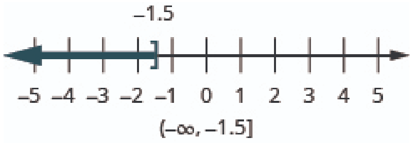

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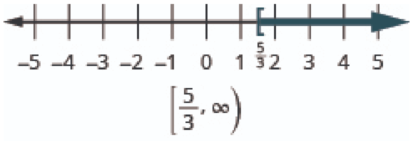


Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

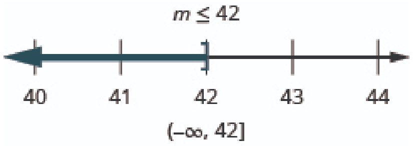
Exercise:

Problem: $n - 12 \le 23$

Exercise:

Problem: $m + 14 \le 56$

Solution:



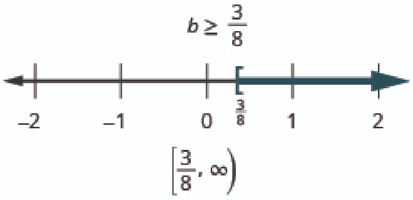
Exercise:

Problem: $a + \frac{2}{3} \ge \frac{7}{12}$

Exercise:

Problem: $b - \frac{7}{8} \ge -\frac{1}{2}$

Solution:



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

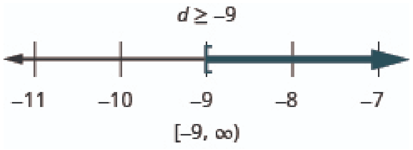
Exercise:

Problem: $9x > 54$

Exercise:

Problem: $-12d \le 108$

Solution:



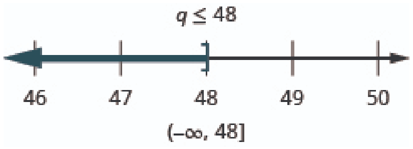
Exercise:

Problem: $\frac{5}{2}j < -60$

Exercise:

Problem: $\frac{q}{-2} \geq -24$

Solution:



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

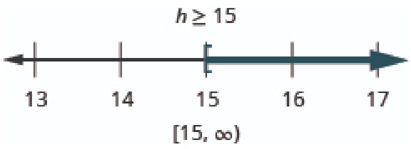
Exercise:

Problem: $6p > 15p - 30$

Exercise:

Problem: $9h - 7(h - 1) \leq 4h - 23$

Solution:



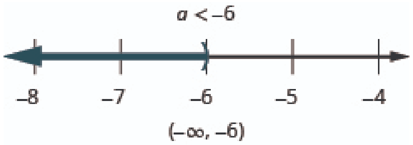
Exercise:

Problem: $5n - 15(4 - n) < 10(n - 6) + 10n$

Exercise:

Problem: $\frac{3}{8}a - \frac{1}{12}a > \frac{5}{12}a + \frac{3}{4}$

Solution:



Translate to an Inequality and Solve

In the following exercises, translate and solve. Then write the solution in interval notation and graph on the number line.

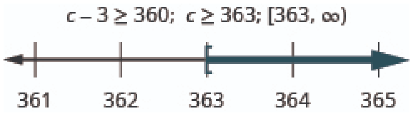
Exercise:

Problem: Five more than *z* is at most 19.

Exercise:

Problem: Three less than *c* is at least 360.

Solution:



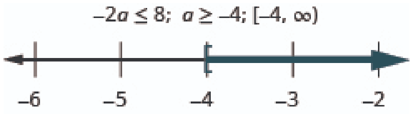
Exercise:

Problem: Nine times *n* exceeds 42.

Exercise:

Problem: Negative two times *a* is no more than 8.

Solution:



Everyday Math

Exercise:

Problem:

Describe how you have used two topics from this chapter in your life outside of your math class during the past month.

Chapter 2 Practice Test

Exercise:

Problem:

Determine whether each number is a solution to the equation $6x-3=x+20$.

ⓐ 5
ⓑ $\frac{23}{5}$

Solution:

ⓐ no ⓑ yes

In the following exercises, solve each equation.

Exercise:

Problem: $n-\frac{2}{3}=\frac{1}{4}$

Exercise:

Problem: $\frac{9}{2}c=144$

Solution:

$c=32$

Exercise:

Problem: $4y-8=16$

Exercise:

Problem: $-8x-15+9x-1=-21$

Solution:

$x = -5$

Exercise:

Problem: $-15a = 120$

Exercise:

Problem: $\frac{2}{3}x = 6$

Solution:

$x = 9$

Exercise:

Problem: $x - 3.8 = 8.2$

Exercise:

Problem: $10y = -5y - 60$

Solution:

$y = -4$

Exercise:

Problem: $8n - 2 = 6n - 12$

Exercise:

Problem: $9m - 2 - 4m - m = 42 - 8$

Solution:

$m = 9$

Exercise:

Problem: $-5(2x - 1) = 45$

Exercise:

Problem: $-(d-9)=23$

Solution:

$d=-14$

Exercise:

Problem: $\frac{1}{4}(12m-28)=6-2(3m-1)$

Exercise:

Problem: $2(6x-5)-8=-22$

Solution:

$x=-\frac{1}{3}$

Exercise:

Problem: $8(3a-5)-7(4a-3)=20-3a$

Exercise:

Problem: $\frac{1}{4}p-\frac{1}{3}=\frac{1}{2}$

Solution:

$p=\frac{10}{3}$

Exercise:

Problem: $0.1d+0.25(d+8)=4.1$

Exercise:

Problem: $14n-3(4n+5)=-9+2(n-8)$

Solution:

contradiction; no solution

Exercise:

Problem: $9(3u-2)-4[6-8(u-1)]=3(u-2)$

Exercise:

Problem: Solve the formula $x-2y=5$ for y
ⓐ when $x=-3$
ⓑ in general

Solution:

ⓐ $y=-4$ ⓑ $y=\frac{5-x}{-2}$

In the following exercises, graph on the number line and write in interval notation.

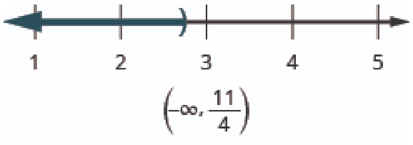
Exercise:

Problem: $x \geq -3.5$

Exercise:

Problem: $x < \frac{11}{4}$

Solution:



In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

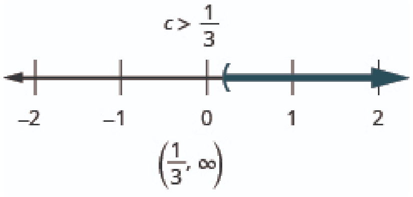
Exercise:

Problem: $8k \geq 5k-120$

Exercise:

Problem: $3c-10(c-2) < 5c+16$

Solution:



In the following exercises, translate to an equation or inequality and solve.

Exercise:

Problem: 4 less than twice *x* is 16.

Exercise:

Problem: Fifteen more than *n* is at least 48.

Solution:

$n + 15 \geq 48; n \geq 33$

Exercise:

Problem:

Samuel paid $25.82 for gas this week, which was $3.47 less than he paid last week. How much had he paid last week?

Exercise:

Problem:

Jenna bought a coat on sale for $120, which was $\frac{2}{3}$ of the original price. What was the original price of the coat?

Solution:

$120 = \frac{2}{3}p$; The original price was $180.

Exercise:

Problem:

Sean took the bus from Seattle to Boise, a distance of 506 miles. If the trip took $7\frac{2}{3}$ hours, what was the speed of the bus?

Exponents, Roots, Factorization of Whole Numbers: Grouping Symbols and the Order of Operations
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses grouping symbols and the order of operations. By the end of the module students should be able to understand the use of grouping symbols, understand and be able to use the order of operations and use the calculator to determine the value of a numerical expression.

Section Overview

- Grouping Symbols
- Multiple Grouping Symbols
- The Order of Operations
- Calculators

Grouping Symbols

Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in mathematics are the following:

( ), [ ], { },
Parentheses: ( )
Brackets: [ ]
Braces: { }
Bar:

In a computation in which more than one operation is involved, grouping symbols indicate which operation to perform first. If possible, we perform operations *inside* grouping symbols first.

Sample Set A

If possible, determine the value of each of the following.

Example:
$9 + (3 \cdot 8)$
Since 3 and 8 are within parentheses, they are to be combined first.

$$\begin{aligned} 9 + (3 \cdot 8) &= 9 + 24 \\ &= 33 \end{aligned}$$

Thus,
$9 + (3 \cdot 8) = 33$

Example:
$(10 \div 0) \cdot 6$
Since $10 \div 0$ is undefined, this operation is meaningless, and we attach no value to it. We write, "undefined."

Practice Set A

If possible, determine the value of each of the following.

Exercise:

Problem: $16-(3\cdot 2)$

Solution:

10

Exercise:

Problem: $5+(7\cdot 9)$

Solution:

68

Exercise:

Problem: $(4+8)\cdot 2$

Solution:

24

Exercise:

Problem: $28\div(18-11)$

Solution:

4

Exercise:

Problem: $(33\div 3)-11$

Solution:

0

Exercise:

Problem: $4+(0\div 0)$

Solution:

not possible (indeterminant)

Multiple Grouping Symbols

When a set of grouping symbols occurs *inside* another set of grouping symbols, we perform the operations within the innermost set first.

Sample Set B

Determine the value of each of the following.

Example:
$2 + (8 \cdot 3) - (5 + 6)$
Combine 8 and 3 first, then combine 5 and 6.
$2 + 24 - 11 \quad \text{Now combine left to right.}$
$26 - 11$
15

Example:
$10 + [30 - (2 \cdot 9)]$
Combine 2 and 9 since they occur in the innermost set of parentheses.
$10 + [30 - 18] \quad \text{Now combine 30 and 18.}$
$10 + 12$
22

Practice Set B

Determine the value of each of the following.
Exercise:

Problem: $(17 + 8) + (9 + 20)$

Solution:

54

Exercise:

Problem: $(55 - 6) - (13 \cdot 2)$

Solution:

23

Exercise:

Problem: $23 + (12 \div 4) - (11 \cdot 2)$

Solution:

4

Exercise:

Problem: $86 + [14 \div (10 - 8)]$

Solution:

93

Exercise:

Problem: $31 + \{9 + [1 + (35 - 2)]\}$

Solution:

74

Exercise:

Problem: $\{6 - [24 \div (4 \cdot 2)]\}^3$

Solution:

27

The Order of Operations

Sometimes there are no grouping symbols indicating which operations to perform first. For example, suppose we wish to find the value of $3 + 5 \cdot 2$. We could do *either* of two things:

Add 3 and 5, then multiply this sum by 2.

$$\begin{aligned} 3 + 5 \cdot 2 &= 8 \cdot 2 \\ &= 16 \end{aligned}$$

Multiply 5 and 2, then add 3 to this product.

$$\begin{aligned} 3 + 5 \cdot 2 &= 3 + 10 \\ &= 13 \end{aligned}$$

We now have two values for one number. To determine the correct value, we must use the *accepted order of operations*.

Order of Operations

1. Perform all operations *inside* grouping symbols, beginning with the innermost set, in the order 2, 3, 4 described below,
2. Perform all exponential and root operations.
3. Perform all multiplications and divisions, moving left to right.
4. Perform all additions and subtractions, moving left to right.

Sample Set C

Determine the value of each of the following.

Example:

$21 + 3 \cdot 12$	Multiply first.
$21 + 36$	Add.
57	

Example:

$(15 - 8) + 5 \cdot (6 + 4)$.	Simplify inside parentheses first.
$7 + 5 \cdot 10$	Multiply.
$7 + 50$	Add.
57	

Example:

$63 - (4 + 6 \cdot 3) + 76 - 4$	Simplify first within the parenthesis by multiplying, then adding.
$63 - (4 + 18) + 76 - 4$	
$63 - 22 + 76 - 4$	Now perform the additions and subtractions, moving left to right.
$41 + 76 - 4$	Add 41 and 76: $41 + 76 = 117$.
$117 - 4$	Subtract 4 from 117: $117 - 4 = 113$.
113	

Example:

$7 \cdot 6 - 4^2 + 1^5$	Evaluate the exponential forms, moving left to right.
$7 \cdot 6 - 16 + 1$	Multiply 7 and 6: $7 \cdot 6 = 42$
$42 - 16 + 1$	Subtract 16 from 42: $42 - 16 = 26$
$26 + 1$	Add 26 and 1: $26 + 1 = 27$
27	

Example:

$6 \cdot (3^2 + 2^2) + 4^2$	Evaluate the exponential forms in the parentheses: $3^2 = 9$ and $2^2 = 4$
$6 \cdot (9 + 4) + 4^2$	Add the 9 and 4 in the parentheses: $9 + 4 = 13$
$6 \cdot (13) + 4^2$	Evaluate the exponential form: $4^2 = 16$
$6 \cdot (13) + 16$	Multiply 6 and 13: $6 \cdot 13 = 78$
$78 + 16$	Add 78 and 16: $78 + 16 = 94$
94	

Example:

$\frac{6^2+2^2}{4^2+6 \cdot 2^2} + \frac{1^3+8^2}{10^2-19 \cdot 5}$	Recall that the bar is a grouping symbol. The fraction $\frac{6^2+2^2}{4^2+6 \cdot 2^2}$ is equivalent to $(6^2 + 2^2) \div (4^2 + 6 \cdot 2^2)$
$\frac{36+4}{16+6 \cdot 4} + \frac{1+64}{100-19 \cdot 5}$	
$\frac{36+4}{16+24} + \frac{1+64}{100-95}$	
$\frac{40}{40} + \frac{65}{5}$	
$1 + 13$	
14	

Practice Set C

Determine the value of each of the following.

Exercise:

Problem: $8 + (32 - 7)$

Solution:

33

Exercise:

Problem: $(34 + 18 - 2 \cdot 3) + 11$

Solution:

57

Exercise:

Problem: $8(10) + 4(2 + 3) - (20 + 3 \cdot 15 + 40 - 5)$

Solution:

0

Exercise:

Problem: $5 \cdot 8 + 4^2 - 2^2$

Solution:

52

Exercise:

Problem: $4(6^2 - 3^3) \div (4^2 - 4)$

Solution:

3

Exercise:

Problem: $(8 + 9 \cdot 3) \div 7 + 5 \cdot (8 \div 4 + 7 + 3 \cdot 5)$

Solution:

125

Exercise:

Problem: $\frac{3^3+2^3}{6^2-29} + 5\left(\frac{8^2+2^4}{7^2-3^2}\right) \div \frac{8 \cdot 3+1^8}{2^3-3}$

Solution:

7

Calculators

Using a calculator is helpful for simplifying computations that involve large numbers.

Sample Set D

Use a calculator to determine each value.

Example:
$9{,}842 + 56 \cdot 85$

	Key		Display Reads
Perform the multiplication first.	Type	56	56
	Press	×	56
	Type	85	85
Now perform the addition.	Press	+	4760
	Type	9842	9842
	Press	=	14602

The display now reads 14,602.

Example:
$42(27+18)+105(810\div 18)$

	Key		Display Reads
Operate inside the parentheses	Type	27	27
	Press	+	27
	Type	18	18
	Press	=	45
Multiply by 42.	Press	×	45
	Type	42	42
	Press	=	1890

Place this result into memory by pressing the memory key.

	Key		Display Reads
Now operate in the other parentheses.	Type	810	810
	Press	÷	810
	Type	18	18
	Press	=	45
Now multiply by 105.	Press	×	45
	Type	105	105
	Press	=	4725
We are now ready to add these two quantities together.	Press	+	4725
Press the memory recall key.			1890
	Press	=	6615

Thus, $42(27+18)+105(810 \div 18)=6{,}615$

Example:
16^4+37^3

Nonscientific Calculators

Key		Display Reads
Type	16	16
Press	×	16
Type	16	16
Press	×	256
Type	16	16
Press	×	4096

Type	16	16
Press	=	65536
Press the memory key		
Type	37	37
Press	×	37
Type	37	37
Press	×	1396
Type	37	37
Press	×	50653
Press	+	50653
Press memory recall key		65536
Press	=	116189

Calculators with y^x Key

Key		Display Reads
Type	16	16
Press	y^x	16
Type	4	4
Press	=	4096
Press	+	4096
Type	37	37
Press	y^x	37
Type	3	3

Press	=	116189

Thus, $16^4 + 37^3 = 116{,}189$
We can certainly see that the more powerful calculator simplifies computations.

Example:
Nonscientific calculators are unable to handle calculations involving very large numbers.
$85612 \cdot 21065$

Key		Display Reads
Type	85612	85612
Press	×	85612
Type	21065	21065
Press	=	

This number is too big for the display of some calculators and we'll probably get some kind of error message. On some scientific calculators such large numbers are coped with by placing them in a form called "scientific notation." Others can do the multiplication directly. (1803416780)

Practice Set D

Use a calculator to find each value.
Exercise:

Problem: $9{,}285 + 86(49)$

Solution:

13,499

Exercise:

Problem: $55(84 - 26) + 120(512 - 488)$

Solution:

6,070

Exercise:

Problem: $106^3 - 17^4$

Solution:

1,107,495

Exercise:

Problem: $6{,}053^3$

Solution:

This number is too big for a nonscientific calculator. A scientific calculator will probably give you 2.217747109×10^{11}

Exercises

For the following problems, find each value. Check each result with a calculator.

Exercise:

Problem: $2 + 3 \cdot (8)$

Solution:

26

Exercise:

Problem: $18 + 7 \cdot (4 - 1)$

Exercise:

Problem: $3 + 8 \cdot (6 - 2) + 11$

Solution:

46

Exercise:

Problem: $1 - 5 \cdot (8 - 8)$

Exercise:

Problem: $37 - 1 \cdot 6^2$

Solution:

1

Exercise:

Problem: $98 \div 2 \div 7^2$

Exercise:

Problem: $(4^2 - 2 \cdot 4) - 2^3$

Solution:

0

Exercise:

Problem: $\sqrt{9} + 14$

Exercise:

Problem: $\sqrt{100} + \sqrt{81} - 4^2$

Solution:

3

Exercise:

Problem: $\sqrt[3]{8} + 8 - 2 \cdot 5$

Exercise:

Problem: $\sqrt[4]{16} - 1 + 5^2$

Solution:

26

Exercise:

Problem: $61 - 22 + 4[3 \cdot (10) + 11]$

Exercise:

Problem: $121 - 4 \cdot [(4) \cdot (5) - 12] + \frac{16}{2}$

Solution:

97

Exercise:

Problem: $\frac{(1+16)-3}{7}+5\cdot(12)$

Exercise:

Problem: $\frac{8\cdot(6+20)}{8}+\frac{3\cdot(6+16)}{22}$

Solution:

29

Exercise:

Problem: $10\cdot[8+2\cdot(6+7)]$

Exercise:

Problem: $21\div7\div3$

Solution:

1

Exercise:

Problem: $10^2\cdot3\div5^2\cdot3-2\cdot3$

Exercise:

Problem: $85\div5\cdot5-85$

Solution:

0

Exercise:

Problem: $\frac{51}{17}+7-2\cdot5\cdot\left(\frac{12}{3}\right)$

Exercise:

Problem: $2^2\cdot3+2^3\cdot(6-2)-(3+17)+11(6)$

Solution:

90

Exercise:

Problem: $26-2\cdot\left\{\frac{6+20}{13}\right\}$

Exercise:

Problem: $2 \cdot \{(7+7)+6 \cdot [4 \cdot (8+2)]\}$

Solution:

508

Exercise:

Problem: $0+10(0)+15 \cdot \{4 \cdot 3+1\}$

Exercise:

Problem: $18+\frac{7+2}{9}$

Solution:

19

Exercise:

Problem: $(4+7) \cdot (8-3)$

Exercise:

Problem: $(6+8) \cdot (5+2-4)$

Solution:

144

Exercise:

Problem: $(21-3) \cdot (6-1) \cdot (7)+4(6+3)$

Exercise:

Problem: $(10+5) \cdot (10+5)-4 \cdot (60-4)$

Solution:

1

Exercise:

Problem: $6 \cdot \{2 \cdot 8+3\}-(5) \cdot (2)+\frac{8}{4}+(1+8) \cdot (1+11)$

Exercise:

Problem: $2^5+3 \cdot (8+1)$

Solution:

52

Exercise:

Problem: $3^4 + 2^4 \cdot (1 + 5)$

Exercise:

Problem: $1^6 + 0^8 + 5^2 \cdot (2 + 8)^3$

Solution:

25,001

Exercise:

Problem: $(7) \cdot (16) - 3^4 + 2^2 \cdot (1^7 + 3^2)$

Exercise:

Problem: $\frac{2^3-7}{5^2}$

Solution:

$\frac{1}{25}$

Exercise:

Problem: $\frac{(1+6)^2+2}{3\cdot 6+1}$

Exercise:

Problem: $\frac{6^2-1}{2^3-3} + \frac{4^3+2\cdot 3}{2\cdot 5}$

Solution:

14

Exercise:

Problem: $\frac{5(8^2-9\cdot 6)}{2^5-7} + \frac{7^2-4^2}{2^4-5}$

Exercise:

Problem: $\frac{(2+1)^3+2^3+1^{10}}{6^2} - \frac{15^2-[2\cdot 5]^2}{5\cdot 5^2}$

Solution:

0

Exercise:

Problem: $\frac{6^3-2\cdot10^2}{2^2}+\frac{18(2^3+7^2)}{2(19)-3^3}$

Exercise:

Problem: $2\cdot\left\{6+\left[10^2-6\sqrt{25}\right]\right\}$

Solution:

152

Exercise:

Problem: $181-3\cdot\left(2\sqrt{36}+3\sqrt[3]{64}\right)$

Exercise:

Problem: $\frac{2\cdot\left(\sqrt{81}-\sqrt[3]{125}\right)}{4^2-10+2^2}$

Solution:

$\frac{4}{5}$

Exercises for Review

Exercise:

Problem:

([link]) The fact that 0 + any whole number = that particular whole number is an example of which property of addition?

Exercise:

Problem: ([link]) Find the product. $4{,}271\times630$.

Solution:

2,690,730

Exercise:

Problem: ([link]) In the statement $27\div3=9$, what name is given to the result 9?

Exercise:

Problem: ([link]) What number is the multiplicative identity?

Solution:

1

Exercise:

Problem: ([link]) Find the value of 2^4.

Box Plots

We have already discussed techniques for visually representing data (see histograms and frequency polygons). In this section we present another important method, called **box plots**. (We encountered a simplified form of box plots in the introduction to this chapter.) Box plots are useful for identifying outliers and for comparing distributions. We will explain box plots with the help of data from an in-class experiment. Students in Introductory Statistics were presented with a page containing 30 colored rectangles. Their task was to name the colors as quickly as possible, and their times were recorded. We'll compare the scores for the 16 men and 31 women who participated in the experiment by making separate box plots for each gender. (Such a display is said to involve **parallel box plots**.)

There are several steps in constructing a box plot. The first relies on the 25th, 50th, and 75th percentiles in the distribution of scores. [link] shows how these three statistics are used. For each gender we draw a box extending from the 25th percentile to the 75th percentile. The 50th percentile is drawn inside the box. Therefore,

```
      the bottom of each box is the 25th
percentile,
      the top is the 75th percentile,
      and the line in the middle is the 50th
percentile.
```

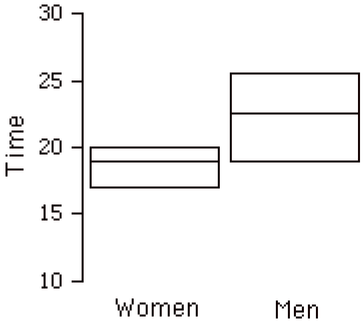
The data for the women in our sample are shown in [link].

14	17	18	19	20	21	29
15	17	18	19	20	21	

16	17	18	19	20	23	
16	17	18	20	20	24	
17	18	18	20	21	24	

Times (in seconds) for women to name the colors.

For these data, the 25th percentile is 17, the 50th percentile is 19, and the 75th percentile is 20. For the men (whose data are not shown), the 25th percentile is 19, the 50th percentile is 22.5, and the 75th percentile is 25.5.



The first step in creating box plots.

Before proceeding, the terminology in [link] is helpful.

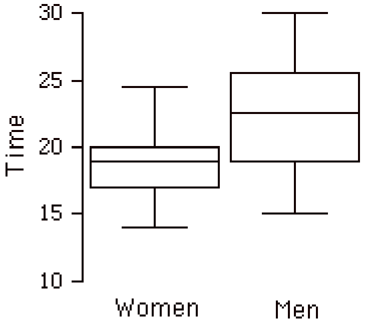
Name	Formula	Value for Women's Data

Name	Formula	Value for Women's Data
Upper Hinge	75th percentile	20
Lower Hinge	25th percentile	17
H-Spread	Upper Hinge − Lower Hinge	3
Step	1.5H-Spread	4.5
Upper Inner Fence	Upper Hinge + 1 Step	24.5
Lower Inner Fence	Lower Hinge − 1 Step	12.5
Upper Outer Fence	Upper Hinge + 2 Steps	29
Lower Outer Fence	Lower Hinge − 2 Steps	8
Upper Adjacent	Largest value below Upper Inner Fence	24
Lower Adjacent	Smallest value above Lower Inner Fence	14

Name	**Formula**	**Value for Women's Data**
Outside Value	A value beyond an Inner Fence but not beyond an Outer Fence	29 (this value is on the fence, but not beyond)
Far Out Value	A value beyond an Outer Fence	None in these data

Terminology

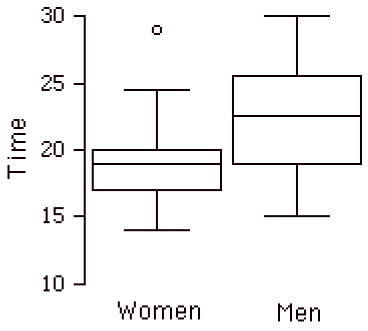
Continuing with the box plots, we put "whiskers" above and below each box, to give additional information about the spread of data ([link]). Whiskers are vertical lines that end in a horizontal stroke (the purpose of the stroke is just to make the vertical lines more visible). Whiskers are drawn from the upper and lower hinges to the upper and lower adjacent values (24 and 14 for the women's data).



The box plots with the whiskers drawn.

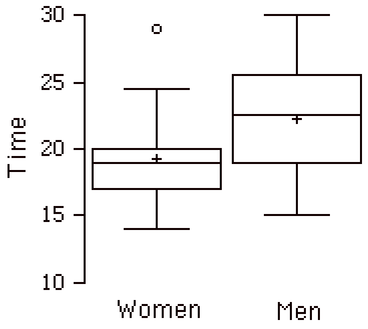
Although we don't draw whiskers all the way to outside or far out values, we still wish to represent these outliers in our box plots. This is achieved by adding additional marks beyond the whiskers. Specifically, outside values are indicated by small circles, and far out values are indicated by asterisks.

In our data, there are no far out values, and just one outside value. The outside value of 29 is for the women, and is shown in [link].



The box plots with the outlier shown.

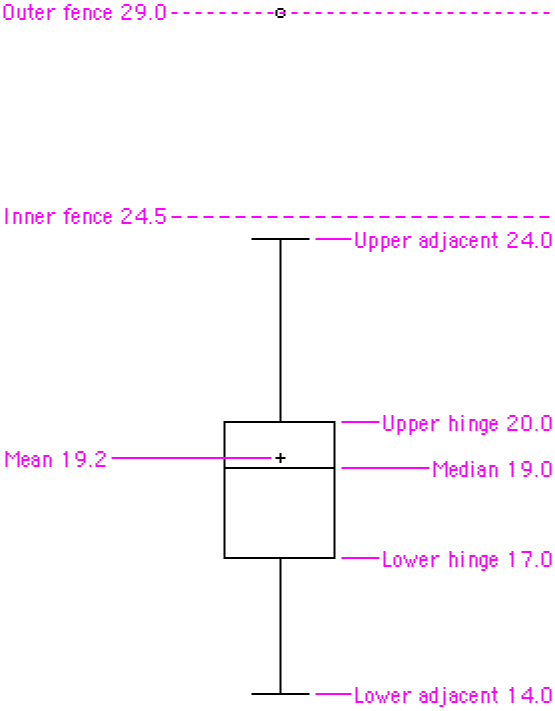
There is one more mark to include in box plots (although sometimes it is omitted). We indicate the mean score for a group by inserting a plus sign. [link] shows the result of adding means to our box plots.



The completed box plots.

[link] provides a revealing summary of the data. Since half the scores in a distribution are between the hinges (recall that the hinges are the 25th and 75th percentiles), we see that half the women's times are between 17 and 20

whereas half the men's times are between 19 and 25. We also see that women generally named the colors faster than the men did, although one woman was slower than almost all of the men. [link] shows the boxplot for the women's data with detailed labels.



The boxplot for the women's data.

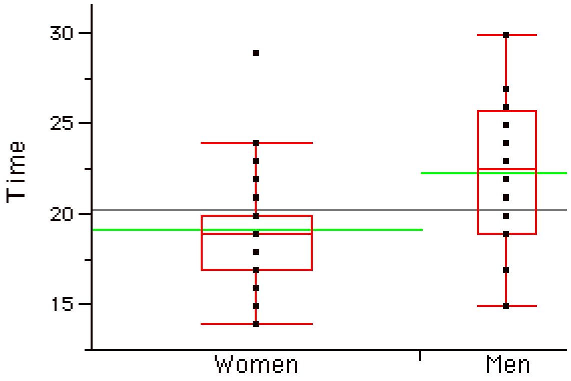
Here are some other examples of box plots.

- Time to move the mouse over a target.
- Draft Lottery

Variations on box plots

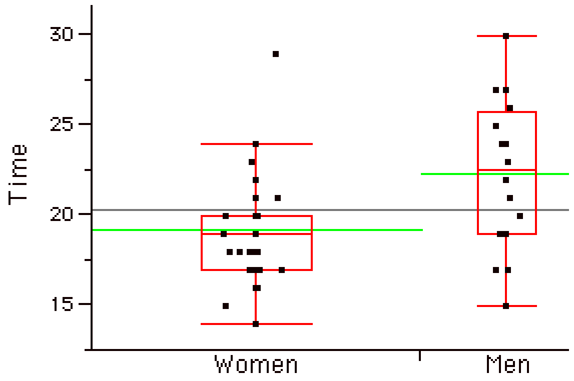
Statistical analysis programs may offer options on how box plots are created. For example, the box plot in [link] is constructed from our data but differs from the previous box plot in several ways.

1. First, it does not mark outliers.
2. Second, the means are indicated by green lines rather than plus signs.
3. The mean of all scores is indicated by a grey line.
4. Individual scores are represented by dots. Since the scores have been rounded to the nearest second, any given dot might represent more than one score.
5. The box for the women is wider than the box for the men because the widths of the boxes are proportional to the number of subjects of each gender (31 women and 16 men).



Box plots showing the individual scores and the means.

Each dot in [link] represents a group of subjects with the same score (rounded to the nearest second). An alternative graphing technique is to **jitter** the points. This means spreading out different dots at the same horizontal position, one dot for each subject. The exact horizontal position of a point is determined randomly (under the constraint that different dots don?t overlap). Spreading out the dots allows you to see multiple occurrences of a given score. [link] shows what jittering looks like.



Box plots with the individual scores jittered.

Different styles of box plots are best for different situations, and there are no firm rules for which to use. When exploring your data you should try several ways of visualizing them. Which graph you include in your report should depend on how well different graphs reveal the aspects of the data you consider most important.

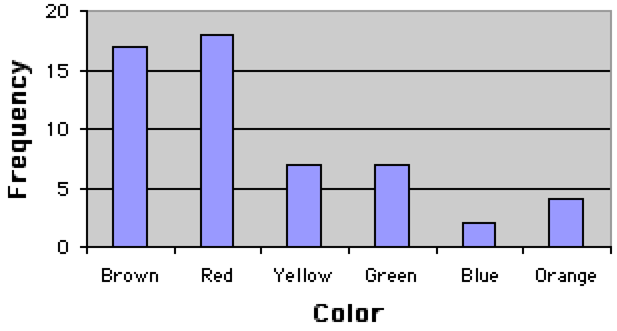
Distributions of Discrete Variables

I recently purchased a bag of Plain M&Ms. The M&M's were in six different colors. A quick count showed that there were 55 M&M's: 17 brown, 18 red, 7 yellow, 7 green, 2 blue, and 4 orange. These counts are shown below in [link].

Color	**Frequency**
Brown	17
Red	18
Yellow	7
Green	7
Blue	2
Orange	4

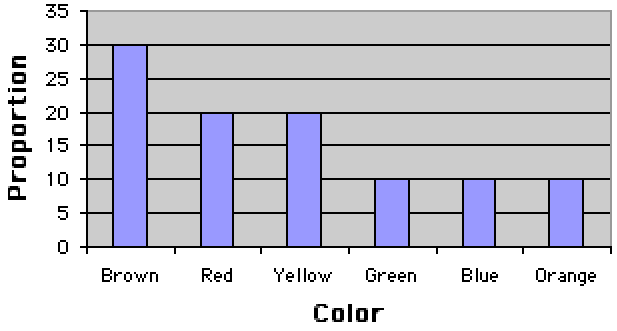
Distributions of Colors

This table is called a **frequency table** and it describes the distribution of M&M color frequencies. Not surprisingly, this kind of table is called a **frequency distribution**. Often a frequency distribution is shown graphically as in [link].



Distribution of 55 M&Ms

The distribution shown in [link] concerns just my one bag of M&M's. You might be wondering about the distribution of colors for all M&M's. The manufacturer of M&M's provides some information about this matter, but they do not tell us exactly how many M&M's of each color they have ever produced. Instead, they report proportions rather than frequencies. [link] shows these proportions. Since every M&M is one of the six familiar colors, the six proportions shown in the figure add to one. We call [link] a **probability distribution** because if you chose an M&M at random, the probability of getting, say, a brown M&M is equal to the proportion of M&M's that are brown (0.30).



Distribution of all M&Ms

Notice that the distributions in [link] and [link] are not identical. [link] portrays the distribution in a sample of 55 M&M's. [link] shows the proportions for all M&M's. Chance factors involving the machines used by the manufacturer introduce random variation into the different bags produced. Some bags will have a distribution of colors that is close to [link]; others will be farther away.

Continuous Variables

The variable "color of M&M" used in this example is a **discrete variable**, and its distributions is also called **discrete**. Let us now extend the concept of a distribution to **continuous variables**.

The data shown in [link] are the times it took one of us (DL) to move the mouse over a small target in a series of 20 trials. The times are sorted from fastest to slowest. The variable "time to respond" is a continuous variable. With time measured accurately (to many decimal places), no two response times would be expected to be the same. Measuring time in milliseconds (thousandths of a second) is often precise enough to approximate a continuous variable in Psychology. As you can see in [link], measuring DL's responses this way produced times no two of which were the same. As a result, a frequency distribution would be uninformative: it would consist of the 20 times in the experiment, each with a frequency of 1.

The solution to this problem is to create a **grouped frequency distribution**. In a grouped frequency distribution, scores falling withing various ranges are tabulated. [link] shows a grouped frequency distribution for these 20 times.

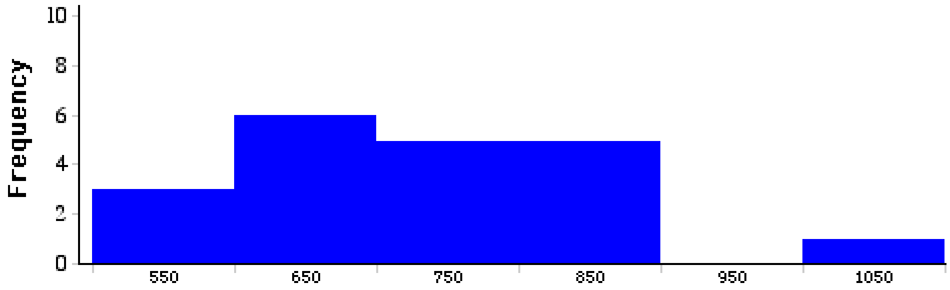
568	720
577	728
581	729
640	777
641	808
645	824
657	825
673	865
696	875
703	1007

Response Times (in milliseconds)

Range	**Frequency**
500-600	3
600-700	6
700-800	5
800-900	5
900-1000	0
1000-1100	1

Grouped frequency distribution

Grouped frequency distributions may be portrayed graphically. [link] shows a graphical representation of the frequency distribution in [link]. This kind of graph is called a **histogram**. Chapter 2 contains an entire section devoted to histograms.

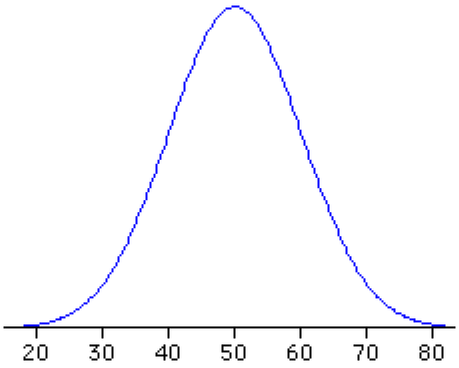


A histogram of the grouped frequency distribution shown in [link]. The labels on the X-axis are the middle values of the range they represent.

Probability Densities

The histogram in [link] portrays just DL's 20 times in the one experiment he performed. To represent the probability associated with an arbitrary movement (which can take any positive amount of time), we must represent all these potential times at once. For this purpose, we plot the distribution for the continuous variable of time. Distributions for continous variables are called **continuous distributions**. They also carry the fancier name **probability density**. Some probability densities have particular importance in Statistics. A very important one is shaped like a bell, and called the **normal distribution**. Many naturally-occuring phenomena can be approximated surprisingly well by this distribution. It will serve to illustrate some features of all continous distributions.

An example of a normal distribution is shown in [link]. Do you see the "bell"? The normal distribution doesn't represent a real bell, however, since the left and right tips extend indefinitely (we can't draw them any further so they look like they've stopped in our diagram). The Y axis in the normal distribution represents the " density of probability." Intuitively, it shows the chance of obtaining values near corresponding points on the X axis. In [link], for example, the probability of an observation with value near 40 is about half of the probability of an observation with value near 50. Although this text does not discuss the concept of probability density in detail, you should keep the following ideas in mind about the curve that describes a continuous distribution (like the normal distribution). First, the area under the curve equals 1. Second, the probabiity of any exact value of X is 0. Finally, the area under the curve and bounded between two given points on the X axis is the probability that a number chosen at random will fall between the two points. Let us illustrate with DL's hand movements. First, the probability that his movement takes some amount of time is one! (We exclude the possibility of him never finishing his gesture.) Second, the probability that his movement takes exactly 598.956432342346576 milliseconds is essentially zero. (We can make the probability as close as we like to zero by making the time measurement more and more precise.) Finally, suppose that the probability of DL's movement taking between 600 and 700 milliseconds is one tenth. Then the continous distribution for DL's possible times would have a shape that places 10% of the area below the curve in the region bounded by 600 and 700 on the X axis.

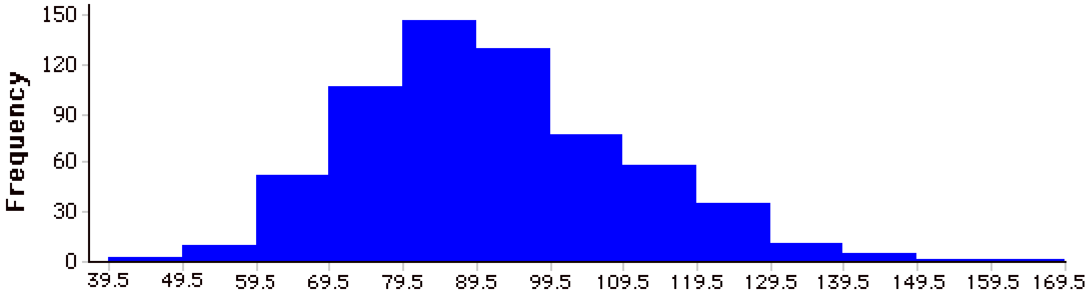


A Normal Distribution

Shapes of Distributions

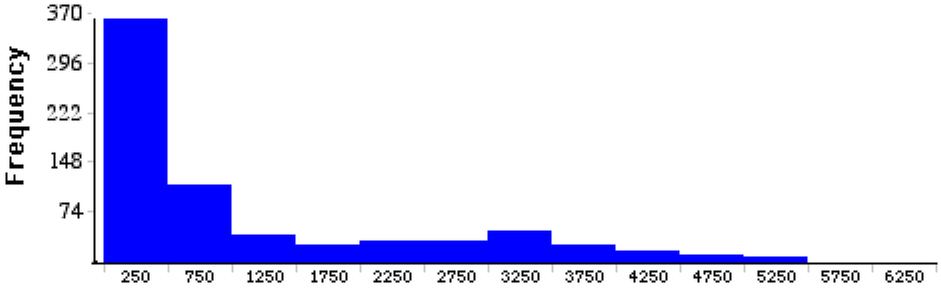
Distributions have different shapes; they don't all look like the normal distribution in [link]. For example, the normal probability density is higher in the middle compared to its two tails. Other distributions need not have this feature. There is even variation among the distributions that we call "normal." For example, some normal distributions are more spread out than the one shown in [link] (their tails begin to hit the X axis further from the middle of the curve --for example, at 10 and 90 if drawn in place of [link]). Others are less spread out (their tails might approach the X axis at 30 and 70). More information on the normal distribution can be found in a later chapter completely devoted to them.

The normal distribution shown in [link] is symmetric; if you folded it in the middle, the two sides would match perfectly. [link] shows the discrete distribution of scores on a psycholoogy test. This distribution is not symmetric: the tail in the positive direction extends further than the tail in the negative direction. A distribution with the longer tail extending in the positive direction is said to have a **positive skew**. It is also described as "skewed to the right."



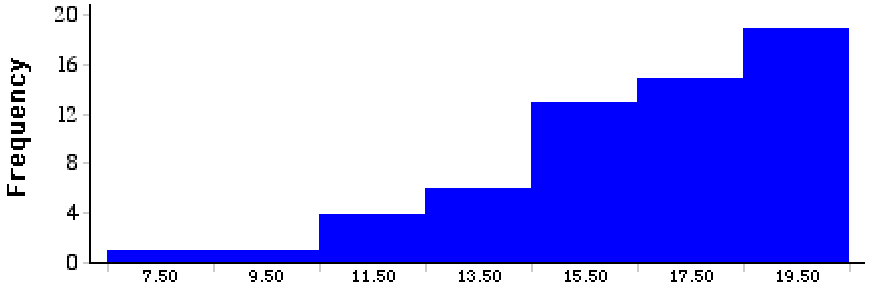
A distribution with a positive skew

[link] shows the salaries of major league baseball players in 1974 (in thousands of dollars). This distribution has an extreme positive skew.



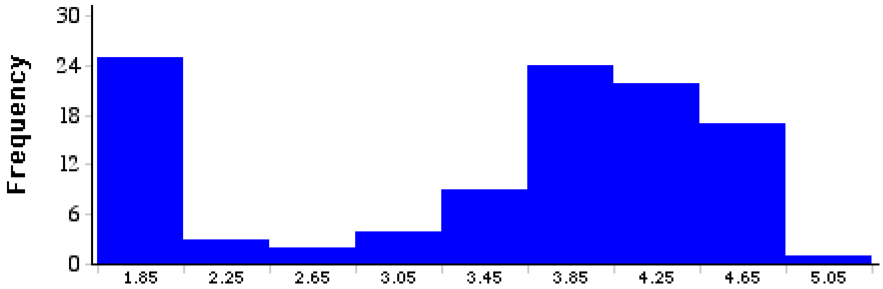
A distribution with a very large positive skew. This histogram shows the salaries of major league baseball players.

Although less common, some distributions have **negative skew**. [link] shows the scores on a 20-point problem on a statistics exam. Since the tail of the disribution extends to the left, this distribution is **skewed to the left**.



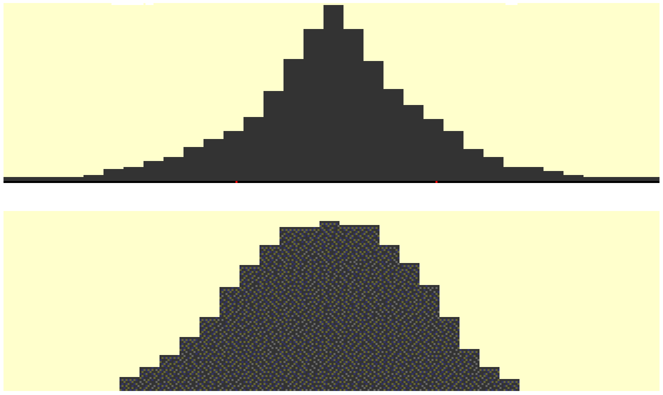
A distribution with negative skew. This histogram shows the frequencies of various scores on a 20-point question on a statistics test.

The distributions shown so far all have one distinct high point or peak. The distribution in [link] has two distinct peaks. A distribution with two peaks is called a **bimodal distribution**.



Frequencies of times between eruptions of the old faithful geyser. Notice the two distinct peaks: one at 1.85 and the other at 3.85.

Distributions also differ from each other in terms of how large or "fat" their tails are. [link] shows two distributions that differ in this respect. The upper distribution has relatively more scores in its tails; its shape is called **leptokurtic**. The lower distribution has relatively fewer scores in its tails; its shape is called **platykurtic**.



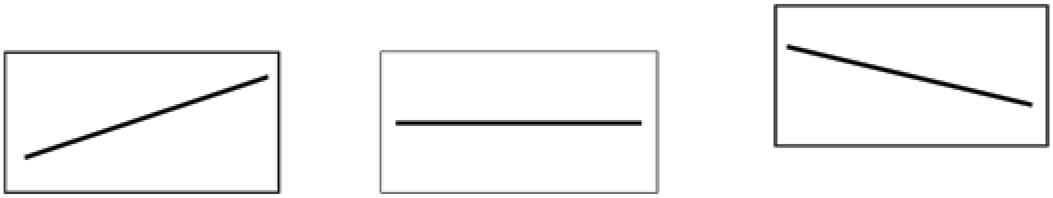
Distributions differing in kurtosis. The top distribution has long tails. It is called "leptokurtic." The bottom distribution has short tails. It is called "platykurtic."

Linear Regression and Correlation: Slope and Y-Intercept of a Linear Equation

This module provides an overview of Linear Regression and Correlation: Slope and Y-Intercept of a Linear Equation as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

For the linear equation $y = a + \mathrm{bx}$, b = slope and a = y-intercept. Note this is similar to $y = \mathrm{mx} + b$, but it is not the same.

From algebra recall that the slope is a number that describes the steepness of a line and the y-intercept is the y coordinate of the point $(0, a)$ where the line crosses the y-axis.



If $b > 0$, the line slopes upward to the right.

If $b = 0$, the line is horizontal.

If $b < 0$, the line slopes downward to the right.

Three possible graphs of $y = a + \mathrm{bx}$.

The slope is how much the response variable will increase/decrease by as the explanatory variable increases by 1. The y-intercept is the value of the response variable when the explanatory variable is 0.

Example:
A
Svetlana tutors to make extra money for college. For each tutoring session, she charges a one time fee of $25 plus $15 per hour of tutoring. A linear

equation that expresses the total amount of money Svetlana earns for each session she tutors is $y = 25 + 15\text{x}$.

Exercise:
A

Problem:

What are the independent and dependent variables? What is the y-intercept and what is the slope? Interpret them using complete sentences.

Solution:

The independent variable (x) is the number of hours Svetlana tutors each session. The dependent variable (y) is the amount, in dollars, Svetlana earns for each session.

The y-intercept is 25 (a = 25). At the start of the tutoring session, Svetlana charges a one-time fee of $25 (this is when x = 0). The slope is 15 (b = 15). For each session, Svetlana earns $15 for each hour she tutors.

Example:
B

The equation to predict your college GPA given your high school GPA is $y = 1.11 + .66\text{x}$,where x is your high school GPA.

Exercise:
B

Problem:

What are the explanatory and response variables? Interpret the slope and y-intercept using complete sentences.

Solution:

The explanatory variable is high school GPA. The response variable is college GPA. The slope is .66; for each additional point on your high school GPA your college GPA should increase by .66 points. The y-intercept is 1.11; a person with a high school GPA of 0 is expected to have a college GPA of 1.1

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